

- Preface p. xvii
- Part A Basic concepts
- 1 Introductory concepts of solid mechanics p. 3
- 1.1 Introduction p. 3
- 1.2 Forces, loads and reactions - idealisations p. 5
- (a) Types of loads p. 5
- (b) Representation of forces and loads p. 6
- (c) Reactions and constraints - idealisations p. 7
- 1.3 Intensity of internal forces - average stresses p. 8
- 1.4 Intensity of a normal force acting over an area - refinement of the concept: normal stress at a point p. 11
- 1.5 Average stresses on an oblique plane p. 12
- 1.6 Variation of internal forces and stresses with position p. 13
- 1.7 Strain as a measure of intensity of deformation p. 14
- 1.8 Mechanical behaviour of materials p. 16
- 1.9 Summary p. 18
- Problems p. 18
- 2 Internal forces and stress p. 21
- 2.1 Introduction p. 21
- 2.2 Internal force resultants p. 21
- 2.3 State of stress at a point: traction p. 27
- (a) Traction p. 27
- (b) Sign convention p. 30
- (c) The stress tensor p. 30
- (d) Equality of the conjugate shear stresses p. 31
- 2.4 Stress equations of motion and equilibrium p. 35
- 2.5 Relations between stress components and internal force resultants p. 38
- 2.6 Stress transformation laws for plane stress p. 42
- (a) Derivation p. 42
- (b) Remarks on the transformation laws (stress as a tensor; invariants of a tensor) p. 46
- (c) Transformation law of a vector: the vector as a tensor p. 47
- 2.7 Principal stresses and stationary shear stress values p. 48
- (a) Principal stresses: stationary values of $[\sigma_{nn}]$ p. 48
- (b) Maximum and minimum shear stress components p. 51
- (c) Summary of results p. 53
- (d) Parametric representation of the state of stress: the Mohr circle p. 53
- 2.8 Cartesian components of traction in terms of stress components: traction on the surface of a body p. 61
- Problems p. 64
- 3 Deformation and strain p. 74
- 3.1 Introduction p. 74
- 3.2 Types of deformation p. 74
- 3.3 Extensional or normal strain p. 75
- 3.4 Shear strain p. 78

- 3.5 Strain-displacement relations p. 81
- (a) Some preliminary instructive examples p. 81
- (b) Strain-displacement relations for infinitesimal strains and rotations p. 85
- 3.6 State of strain p. 92
- 3.7 Two-dimensional transformation law for infinitesimal strain components p. 93
- (a) Geometric derivation p. 93
- (b) Analytic derivation of the transformation laws p. 96
- (c) The infinitesimal strain tensor - two-dimensional transformation laws p. 98
- 3.8 Principal strains and principal directions of strain: the Mohr circle for strain p. 100
- 3.9 The strain rosette p. 105
- 3.10 Volumetric strain - dilatation p. 107
- Problems p. 108
- 4 Behaviour of materials: constitutive equations p. 119
- 4.1 Introduction p. 119
- 4.2 Some general idealisations (definitions: 'micro' and 'macro' scales) p. 119
- 4.3 Classification of materials: viscous, elastic, visco-elastic and plastic materials p. 121
- 4.4 Elastic materials p. 123
- (a) Constitutive equations for elastic materials: general elastic and linear elastic behaviour, Hooke's law p. 123
- (b) Elastic strain energy p. 130
- 4.5 Mechanical properties of engineering materials p. 134
- (a) Behaviour of ductile materials p. 134
- (b) Behaviour of brittle materials p. 139
- (c) Behaviour of rubber-like materials p. 140
- 4.6 Plastic behaviour: idealised models p. 141
- Problems p. 142
- 5 Summary of basic results and further idealisations: solutions using the 'mechanics-of-materials' approach p. 145
- 5.1 Introduction p. 145
- 5.2 Superposition principles p. 147
- (a) Superposition of infinitesimal strains p. 147
- (b) Basic principle of superposition for linear elastic bodies p. 148
- 5.3 The principle of de Saint Venant p. 150
- Part B Applications to simple elements
- 6 Axial loadings p. 155
- 6.1 Introduction p. 155
- 6.2 Elastic behaviour of prismatic rods: basic results p. 155
- 6.3 Some general comments p. 159
- 6.4 Extension of results: approximations for rods having varying cross-sections p. 162
- 6.5 Statically indeterminate axially loaded members p. 164
- 6.6 Temperature problems: thermal stresses p. 170
- 6.7 Elastic-plastic behaviour: residual stresses p. 174
- Problems p. 179
- 7 Torsion of circular cylindrical rods: Coulomb torsion p. 190

- 7.1 Introduction p. 190
- 7.2 Basic relations for elastic members under pure torsion p. 190
- (a) Deformation analysis: conclusions based on axi-symmetry of the rod p. 190
- (b) Basic relations p. 193
- 7.3 Some comments on the derived expressions: extension of the results and approximations p. 197
- (a) Comments on the solution p. 197
- (b) An approximation for thin-wall circular tubular cross-sections p. 200
- (c) Extension of the results: engineering approximations p. 201
- 7.4 Some practical engineering design applications of the theory p. 204
- 7.5 Circular members under combined loads p. 206
- 7.6 Statically indeterminate systems under torsion p. 207
- 7.7 Elastic-plastic torsion p. 210
- Problems p. 213
- 8 Symmetric bending of beams - basic relations and stresses p. 225
- 8.1 Introduction p. 225
- 8.2 Resultant shear and bending moments - sign convention p. 225
- (a) Some simple examples p. 225
- (b) Sign convention p. 229
- 8.3 Differential relations for beams p. 230
- 8.4 Some further examples for resultant forces in beams p. 231
- 8.5 Integral relations for beams p. 237
- 8.6 Symmetrical bending of beams in a state of pure bending p. 242
- (a) Some preliminary definitions and limitations - deformation analysis p. 242
- (b) Moment-curvature relations and flexural stresses in an elastic beam under pure bending: Euler-Bernoulli relations p. 244
- (c) Axial displacements of beams under pure bending p. 252
- (d) Comments on the solution - exactness of the solution p. 254
- (e) Methodology of solution - the methodology of 'mechanics of materials' p. 254
- 8.7 Flexure of beams due to applied lateral loads - Navier's hypothesis p. 255
- 8.8 Shear stresses in beams due to symmetric bending p. 259
- (a) Derivation p. 259
- (b) Limitations on the derived expression p. 264
- (c) Shear effect on beams - warping of the cross-sections due to shear p. 265
- 8.9 Re-examination of the expression for flexural stress $[\sigma_x] = My/I$: further engineering approximations p. 265
- (a) Examination of equilibrium state p. 265
- (b) Flexural stress in a non-prismatic beam - an engineering approximation p. 267
- 8.10 Engineering design applications for beams p. 269
- 8.11 Bending of composite beams p. 272
- 8.12 Combined loads p. 276
- 8.13 Elastic-plastic behaviour p. 278
- (a) Fully plastic moments - location of the neutral axis p. 278

- (b) Moment-curvature relation for beams of rectangular cross-section in the plastic range p. 282
- Problems p. 286
- 9 Symmetric bending of beams: deflections, fundamental solutions and superposition p. 313
- 9.1 Introduction p. 313
- 9.2 Linearised beam theory p. 314
- 9.3 Accuracy of the linearised beam theory p. 316
- 9.4 Elastic curve equations for some 'classical' cases p. 318
- 9.5 Axial displacements due to flexure of a beam under lateral loads p. 324
- 9.6 Deflections due to shear deformation p. 326
- 9.7 Singularity functions and their application p. 329
- (a) Definition of singularity functions p. 329
- (b) Applications p. 330
- 9.8 Solutions for statically indeterminate beams by integration of the differential equation p. 337
- 9.9 Application of linear superposition in beam theory p. 339
- 9.10 Analysis of statically indeterminate beams: the force method p. 342
- (a) Development of the force method p. 342
- (b) Comments on the force method p. 348
- 9.11 Superposition - integral formulation: the fundamental solution and Green's functions p. 349
- (a) Development and applications p. 349
- (b) Generalisation: Green's functions for shears, moments, etc. in beams p. 352
- (c) Some general comments p. 356
- 9.12 The fourth-order differential equation for beams p. 357
- (a) Development and applications p. 357
- (b) The fourth-order differential equation for concentrated force and couple loadings p. 359
- 9.13 Moment-area theorems p. 365
- Problems p. 375
- 10 Thin-wall pressure vessels: thin shells under pressure p. 392
- 10.1 Introduction p. 392
- 10.2 Thin cylindrical shells p. 392
- 10.3 Thin spherical shells p. 397
- 10.4 Comments and closure p. 398
- Problems p. 399
- 11 Stability and instability of rods under axial compression: beam-columns and tie-rods p. 404
- 11.1 Introduction p. 404
- 11.2 Stability and instability of mechanical systems p. 405
- 11.3 Stability of rigid rods under compressive loads: the concept of bifurcation p. 406
- 11.4 Stability of an elastic rod subjected to an axial compressive force-Euler buckling load p. 411
- 11.5 Elastic buckling of rods under various boundary conditions p. 415
- 11.6 Rods under eccentric axial loads-the 'secant formula' p. 420

- 11.7 Rods under combined axial and lateral loads: preliminary remarks p. 423
- 11.8 Differential equations of beams subjected to combined lateral loads and axial forces p. 424
- 11.9 Stability analysis using the fourth-order differential equation p. 426
- 11.10 Beam-column subjected to a single lateral force F and an axial compressive force P p. 428
- 11.11 Some comments on the solution: use of linear superposition p. 432
- 11.12 Tie-rods p. 433
- 11.13 General comments and conclusions p. 435
- Problems p. 436
- 12 Torsion of elastic members of arbitrary cross-section: de Saint Venant torsion p. 447
- 12.1 Introduction p. 447
- 12.2 Semi-inverse methods: uniqueness of solutions p. 448
- 12.3 The general de Saint Venant torsion solution p. 449
- 12.4 Torsion of a member of elliptic cross-section p. 458
- 12.5 Torsion of a member of rectangular cross-section p. 462
- 12.6 The membrane analogy p. 468
- 12.7 Torsion of a member having a narrow rectangular cross-section p. 472
- (a) Derivation of membrane analogy solution p. 472
- (b) Comparison of exact solution with membrane analogy for narrow rectangular sections p. 474
- 12.8 Torsion of thin-wall open-section members p. 475
- 12.9 Shear stress at a re-entrant corner: approximate solution p. 478
- 12.10 Torsion of closed-section members: thin-wall sections p. 481
- 12.11 Torsion of multi-cell closed thin-wall sections p. 487
- 12.12 Closure p. 489
- Problems p. 490
- 13 General bending theory of beams p. 496
- 13.1 Introduction p. 496
- 13.2 Moment-curvature relation for elastic beams in flexure p. 497
- 13.3 Sign convention and beam equations for bending about two axes p. 500
- (a) Sign convention p. 500
- (b) Differential beam equations p. 501
- 13.4 General expression for stresses due to flexure p. 502
- (a) Derivation: stresses in beams under pure bending p. 502
- (b) Extension of expression for flexural stress in beams due to applied lateral loads p. 504
- (c) Some particular cases p. 504
- (d) General case p. 506
- 13.5 Shear stresses due to bending of beams p. 508
- (a) Derivation p. 508
- (b) Comments on the expressions p. 511
- 13.6 Distribution of shear stresses in a thin-wall section: shear centers p. 513
- (a) Shear stress distribution p. 513
- (b) The shear center p. 514

- (c) Some remarks and comments p. 517
- 13.7 Deflections and rotations of a beam under applied loads p. 518
- 13.8 Shear stresses in closed thin-wall sections p. 520
- Problems p. 525
- Part C Energy methods and virtual work
- 14 Basic energy theorems, principles of virtual work and their applications to structural mechanics p. 537
 - 14.1 Introduction p. 537
 - 14.2 Elastic strain energy p. 537
 - (a) Review of results for the uniaxial state of stress p. 538
 - (b) General stress state p. 539
 - (c) Examples of strain energy for linear elastic bodies p. 541
 - 14.3 The principle of conservation of energy for linear elastic bodies p. 542
 - (a) Derivation of the principle p. 542
 - (b) Application of the principle p. 545
 - 14.4 Betti's law and Maxwell's reciprocal relation: flexibility coefficients p. 546
 - 14.5 Castigliano's second theorem p. 550
 - 14.6 Geometric representation (complementary strain energy and Castigliano's first theorem) p. 556
 - 14.7 The principle of virtual work p. 558
 - (a) Introduction p. 558
 - (b) Definitions of external and internal virtual work: virtual displacements p. 558
 - (c) Proof of the principle of virtual work: comments on the principle p. 562
 - (d) The principle of virtual work for flexure of beams p. 567
 - (e) Application of the principle of virtual work to evaluate reactions and internal stress resultants: the 'method of virtual displacements' p. 570
 - (f) Influence lines for reactions, shears and moments in beams by the principle of virtual work p. 588
 - 14.8 The principle of complementary virtual work p. 600
 - (a) Introduction p. 600
 - (b) Development and derivation of the principle p. 600
 - (c) Comparison and analogues between the two principles p. 605
 - (d) Expressions for internal complementary virtual work in terms of internal stress resultants: generalised forces and displacements p. 607
 - (e) Internal complementary virtual work in linear elastic rods and beams: explicit expressions (some generalisations) p. 613
 - (f) Application of the principle of complementary virtual work to evaluate displacements of linear elastic bodies: the 'method of virtual forces' p. 615
 - 14.9 The principle of stationary potential energy p. 623
 - (a) Derivation of the principle and some applications p. 623
 - (b) Approximate solutions-the Rayleigh-Ritz method p. 629
 - 14.10 Summary and conclusions p. 634
 - Problems p. 634
- 15 Stability of mechanical systems by energy considerations: approximate methods p. 646

- 15.1 Introduction p. 646
- 15.2 Classification of equilibrium states according to energy criteria p. 646
- 15.3 Stability of a rigid rod subjected to a compressive axial force p. 648
- 15.4 Determination of critical loads using a small deflection analysis-pseudo-neutral equilibrium p. 655
- 15.5 The total potential for small displacements: reconsideration of the stability criteria p. 659
- 15.6 Systems having several degrees-of-freedom-small displacement analysis p. 661
- (a) Two-degree-of-freedom system p. 661
- (b) n-Degree-of-freedom systems p. 665
- 15.7 Stability of an elastic rod: the Rayleigh quotient p. 666
- 15.8 The Rayleigh method for critical loads p. 671
- (a) Development of the method p. 671
- (b) Proof of the upper boundedness of the Rayleigh load (restricted proof) p. 677
- 15.9 The Rayleigh-Ritz method for critical loads p. 679
- Problems p. 682
- Appendix A Properties of areas p. 687
- A.1 General properties: centroids, first and second moments of areas p. 687
- A.2 Properties of selected areas p. 692
- Appendix B Some mathematical relations p. 694
- B.1 Curvature of a line $y = y(x)$ p. 694
- B.2 Green's theorem p. 695
- B.3 The divergence theorem (Gauss' theorem) p. 696
- Appendix C The membrane equation p. 698
- Appendix D Material properties p. 701
- Appendix E Table of structural properties p. 703
- Appendix F Reactions, deflections and slopes of selected beams p. 710
- Answers to selected problems p. 715
- Index p. 723