- 1 Signal Representation p. 1
- 1.1 Introduction p. 1
- 1.2 Why Do We Discretize Continuous Systems? p. 2
- 1.3 Periodic and Nonperiodic Discrete Signals p. 3
- 1.4 The Unit Step Discrete Signal p. 4
- 1.5 The Impulse Discrete Signal p. 6
- 1.6 The Ramp Discrete Signal p. 6
- 1.7 The Real Exponential Discrete Signal p. 7
- 1.8 The Sinusoidal Discrete Signal p. 7
- 1.9 The Exponentially Modulated Sinusoidal Signal p. 11
- 1.10 The Complex Periodic Discrete Signal p. 11
- 1.11 The Shifting Operation p. 15
- 1.12 Representing a Discrete Signal Using Impulses p. 16
- 1.13 The Reflection Operation p. 18
- 1.14 Time Scaling p. 19
- 1.15 Amplitude Scaling p. 20
- 1.16 Even and Odd Discrete Signal p. 21
- 1.17 Does a Discrete Signal Have a Time Constant? p. 23
- 1.18 Basic Operations on Discrete Signals p. 25
- 1.18.1 Modulation p. 25
- 1.18.2 Addition and Subtraction p. 25
- 1.18.3 Scalar Multiplication p. 25
- 1.18.4 Combined Operations p. 26
- 1.19 Energy and Power Discrete Signals p. 28
- 1.20 Bounded and Unbounded Discrete Signals p. 30
- 1.21 Some Insights: Signals in the Real World p. 30
- 1.21.1 The Step Signal p. 31
- 1.21.2 The Impulse Signal p. 31
- 1.21.3 The Sinusoidal Signal p. 31
- 1.21.4 The Ramp Signal p. 31
- 1.21.5 Other Signals p. 32
- 1.22 End of Chapter Examples p. 32
- 1.23 End of Chapter Problems p. 50
- 2 The Discrete System p. 55
- 2.1 Definition of a System p. 55
- 2.2 Input and Output p. 55
- 2.3 Linear Discrete Systems p. 56
- 2.4 Time Invariance and Discrete Signals p. 58
- 2.5 Systems with Memory p. 60
- 2.6 Causal Systems p. 61
- 2.7 The Inverse of a System p. 62
- 2.8 Stable System p. 63
- 2.9 Convolution p. 64
- 2.10 Difference Equations of Physical Systems p. 68
- 2.11 The Homogeneous Difference Equation and Its Solution p. 69
- 2.11.1 Case When Roots Are All Distinct p. 71

- 2.11.2 Case When Two Roots Are Real and Equal p. 72
- 2.11.3 Case When Two Roots Are Complex p. 72
- 2.12 Nonhomogeneous Difference Equations and their Solutions p. 73
- 2.12.1 How Do We Find the Particular Solution? p. 75
- 2.13 The Stability of Linear Discrete Systems: The Characteristic Equation p. 75
- 2.13.1 Stability Depending On the Values of the Poles p. 75
- 2.13.2 Stability from the Jury Test p. 76
- 2.14 Block Diagram Representation of Linear Discrete Systems p. 78
- 2.14.1 The Delay Element p. 79
- 2.14.2 The Summing/Subtracting Junction p. 79
- 2.14.3 The Multiplier p. 79
- 2.15 From the Block Diagram to the Difference Equation p. 81
- 2.16 From the Difference Equation to the Block Diagram: A Formal Procedure p. 82
- 2.17 The Impulse Response p. 85
- 2.18 Correlation p. 87
- 2.18.1 Cross-Correlation p. 87
- 2.18.2 Auto-Correlation p. 89
- 2.19 Some Insights p. 90
- 2.19.1 How Can We Find These Eigenvalues? p. 90
- 2.19.2 Stability and Eigenvalues p. 91
- 2.20 End of Chapter Examples p. 91
- 2.21 End of Chapter Problems p. 134
- 3 The Fourier Series and the Fourier Transform of Discrete Signals p. 143
- 3.1 Introduction p. 143
- 3.2 Review of Complex Numbers p. 143
- 3.2.1 Definition p. 145
- 3.2.2 Addition p. 145
- 3.2.3 Subtraction p. 145
- 3.2.4 Multiplication p. 145
- 3.2.5 Division p. 146
- 3.2.6 From Rectangular to Polar p. 146
- 3.2.7 From Polar to Rectangular p. 146
- 3.3 The Fourier Series of Discrete Periodic Signals p. 147
- 3.4 The Discrete System with Periodic Inputs: The Steady-State Response p. 150
- 3.4.1 The General Form for yss(n) p. 153
- 3.5 The Frequency Response of Discrete Systems p. 154
- 3.5.1 Properties of the Frequency Response p. 157
- 3.5.1.1 The Periodicity Property p. 157
- 3.5.1.2 The Symmetry Property p. 157
- 3.6 The Fourier Transform of Discrete Signals p. 159
- 3.7 Convergence Conditions p. 161
- 3.8 Properties of the Fourier Transform of Discrete Signals p. 162
- 3.8.1 The Periodicity Property p. 162
- 3.8.2 The Linearity Property p. 162
- 3.8.3 The Discrete-Time Shifting Property p. 163
- 3.8.4 The Frequency Shifting Property p. 163

- 3.8.5 The Reflection Property p. 163
- 3.8.6 The Convolution Property p. 164
- 3.9 Parseval's Relation and Energy Calculations p. 167
- 3.10 Numerical Evaluation of the Fourier Transform of Discrete Signals p. 168
- 3.11 Some Insights: Why Is This Fourier Transform? p. 172
- 3.11.1 The Ease in Analysis and Design p. 172
- 3.11.2 Sinusoidal Analysis p. 173
- 3.12 End of Chapter Examples p. 173
- 3.13 End of Chapter Problems p. 189
- 4 The z-Transform and Discrete Systems p. 195
- 4.1 Introduction p. 195
- 4.2 The Bilateral z-Transform p. 195
- 4.3 The Unilateral z-Transform p. 197
- 4.4 Convergence Considerations p. 200
- 4.5 The Inverse z-Transform p. 203
- 4.5.1 Partial Fraction Expansion p. 203
- 4.5.2 Long Division p. 206
- 4.6 Properties of the z-Transform p. 207
- 4.6.1 Linearity Property p. 207
- 4.6.2 Shifting Property p. 207
- 4.6.3 Multiplication by e[superscript -an] p. 209
- 4.6.4 Convolution p. 210
- 4.7 Representation of Transfer Functions as Block Diagrams p. 210
- 4.8 x(n), h(n), y(n), and the z-Transform p. 212
- 4.9 Solving Difference Equation using the z-Transform p. 214
- 4.10 Convergence Revisited p. 216
- 4.11 The Final Value Theorem p. 219
- 4.12 The Initial-Value Theorem p. 219
- 4.13 Some Insights: Poles and Zeroes p. 220
- 4.13.1 The Poles of the System p. 220
- 4.13.2 The Zeros of the System p. 221
- 4.13.3 The Stability of the System p. 221
- 4.14 End of Chapter Exercises p. 221
- 4.15 End of Chapter Problems p. 255
- 5 State-Space and Discrete Systems p. 265
- 5.1 Introduction p. 265
- 5.2 A Review on Matrix Algebra p. 266
- 5.2.1 Definition, General Terms and Notations p. 266
- 5.2.2 The Identity Matrix p. 266
- 5.2.3 Adding Two Matrices p. 267
- 5.2.4 Subtracting Two Matrices p. 267
- 5.2.5 Multiplying A Matrix by a Constant p. 267
- 5.2.6 Determinant of a Two-by-Two Matrix p. 268
- 5.2.7 Transpose of A Matrix p. 268
- 5.2.8 Inverse of A Matrix p. 268
- 5.2.9 Matrix Multiplication p. 269

- 5.2.10 Eigenvalues of a Matrix p. 269
- 5.2.11 Diagonal Form of a Matrix p. 269
- 5.2.12 Eigenvectors of a Matrix p. 269
- 5.3 General Representation of Systems in State-Space p. 270
- 5.3.1 Recursive Systems p. 270
- 5.3.2 Nonrecursive Systems p. 272
- 5.3.3 From the Block Diagram to State-Space p. 273
- 5.3.4 From the Transfer Function H(z) to State-Space p. 276
- 5.4 Solution of the State-Space Equations in the z-Domain p. 283
- 5.5 General Solution of the State Equation in Real-Time p. 284
- 5.6 Properties of A[superscript n] and Its Evaluation p. 285
- 5.7 Transformations for State-Space Representations p. 289
- 5.10 End of Chapter Problems p. 322
- 5.8 Some Insights: Poles and Stability p. 291
- 5.9 End of Chapter Examples p. 292
- 6 Modeling and Representation of Discrete Linear Systems p. 329
- 6.1 Introduction p. 329
- 6.2 Five Ways of Representing Discrete Linear Systems p. 330
- 6.2.1 From the Difference Equation to the Other Four Representations p. 330
- 6.2.1.1 The Difference Equation Representation p. 330
- 6.2.1.2 The Impulse Response Representation p. 331
- 6.2.1.3 The z-Transform Representation p. 332
- 6.2.1.4 The State-Space Representation p. 333
- 6.2.1.5 The Block Diagram Representation p. 334
- 6.2.2 From the Impulse Response to the Other Four Representations p. 335
- 6.2.2.1 The Impulse Response Representation p. 335
- 6.2.2.2 The Transfer Function Representation p. 335
- 6.2.2.3 The Different Equation Representation p. 336
- 6.2.2.4 The State-Space Representation p. 336
- 6.2.2.5 The Block Diagram Representation p. 337
- 6.2.3 From the Transfer Function to the Other Four Representations p. 337
- 6.2.3.1 The Transfer Function Representation p. 337
- 6.2.3.2 The Impulse Response Representation p. 338
- 6.2.3.3 The Difference Equation Representation p. 338
- 6.2.3.4 The State-Space Representation p. 339
- 6.2.3.5 The Block Diagram Representation p. 339
- 6.2.4 From the State-Space to the Other Four Representations p. 340
- 6.2.4.1 The State-Space Representation p. 340
- 6.2.4.2 The Transfer Function Representation p. 340
- 6.2.4.3 The Impulse Response Representation p. 341
- 6.2.4.4 The Difference Equation Representation p. 341
- 6.2.4.5 The Block Diagram Representation p. 342
- 6.2.5 From the Block Diagram to the Other Four Representations p. 343
- 6.2.5.1 The State-Space Representation p. 343
- 6.2.5.2 The Transfer Function Representation p. 344
- 6.2.5.3 The Impulse Response Representation p. 345

- 6.2.5.4 The Difference Equation Representation p. 345
- 6.3 Some Insights: The Poles Considering Different Outputs within the Same System p. 346
- 6.4 End of Chapter Exercises p. 346
- 6.5 End of Chapter Problems p. 361
- 7 The Discrete Fourier Transform and Discrete Systems p. 365
- 7.1 Introduction p. 365
- 7.2 The Discrete Fourier Transform and the Finite-Duration Discrete Signals p. 366
- 7.3 Properties of the Discrete Fourier Transform p. 367
- 7.3.1 How Does the Defining Equation Work? p. 367
- 7.3.2 The DFT Symmetry p. 368
- 7.3.3 The DFT Linearity p. 370
- 7.3.4 The Magnitude of the DFT p. 371
- 7.3.5 What Does k in X(k), the DFT, Mean? p. 372
- 7.4 The Relation the DFT Has with the Fourier Transform of Discrete Signals, the z-Transform and the Continuous Fourier Transform p. 373
- 7.4.1 The DFT and the Fourier Transform of x(n) p. 373
- 7.4.2 The DFT and the z-Transform of x(n) p. 374
- 7.4.3 The DFT and the Continuous Fourier Transform of x(t) p. 376
- 7.5 Numerical Computation of the DFT p. 377
- 7.6 The Fast Fourier Transform: A Faster Way of Computing the DFT p. 378
- 7.7 Applications of the DFT p. 380
- 7.7.1 Circular Convolution p. 380
- 7.7.2 Linear Convolution p. 384
- 7.7.3 Approximation to the Continuous Fourier Transform p. 385
- 7.7.4 Approximation to the Coefficients of the Fourier Series and the Average Power of the Periodic Signal x(t) p. 387
- 7.7.5 Total Energy in the Signal x(n) and x(t) p. 391
- 7.7.6 Block Filtering p. 393
- 7.7.7 Correlation p. 395
- 7.8 Some Insights p. 395
- 7.8.1 The DFT Is the Same as the fft p. 395
- 7.8.2 The DFT Points Are the Samples of the Fourier Transform of x(n) p. 395
- 7.8.3 How Can We Be Certain That Most of the Frequency Contents of x(t) Are in the DFT? p. 395
- 7.8.4 Is the Circular Convolution the Same as the Linear Convolution? p. 396
- 7.8.5 Is [vertical bar]X(w)[vertical bar] [congruent with] [vertical bar]X(k)[vertical bar]? p. 396
- 7.8.6 Frequency Leakage and the DFT p. 396
- 7.9 End of Chapter Exercises p. 396
- 7.10 End of Chapter Problems p. 415
- 8 Analogue Filter Design p. 421
- 8.1 Introduction p. 421
- 8.2 Analogue Filter Specifications p. 422
- 8.3 Butterworth Filter Approximation p. 425
- 8.4 Chebyshev Filters p. 428

- 8.6 Bessel Filters p. 434
- 8.4.1 Type I Chebyshev Approximation p. 428
- 8.4.2 Inverse Chebyshev Filter (Type II Chebyshev Filters) p. 431
- 8.5 Elliptic Filter Approximation p. 433
- 8.7 Analogue Frequency Transformation p. 437
- 8.8 Analogue Filter Design using Matlab p. 438
- 8.8.1 Order Estimation Functions p. 439
- 8.8.2 Analogue Prototype Design Functions p. 440
- 8.10 Limitations p. 447
- 8.8.3 Complete Classical IIR Filter Design p. 440
- 8.8.4 Analogue Frequency Transformation p. 442
- 8.9 How Do We Find the Cut-Off Frequency Analytically? p. 443
- 8.11 Comparison between Analogue Filter Types p. 447
- 8.12 Some Insights: Filters with High Gain vs. Filters with Low Gain and the Relation between the Time Constant and the Cut-Off Frequency for First-Order Circuits and the Series RLC Circuit p. 448
- 8.13 End of Chapter Examples p. 449
- 8.14 End of Chapter Problems p. 479
- 9 Transformations between Continuous and Discrete Representations p. 487
- 9.1 The Need for Converting Continuous Signal to a Discrete Signal p. 487
- 9.2 From the Continuous Signal to Its Binary Code Representation p. 488
- 9.3 From the Binary Code to the Continuous Signal p. 490
- 9.4 The Sampling Operation p. 490
- 9.4.1 Ambiguity in Real-Time Domain p. 490
- 9.4.2 Ambiguity in the Frequency Domain p. 492
- 9.4.3 The Sampling Theorem p. 493
- 9.4.4 Filtering before Sampling p. 494
- 9.4.5 Sampling and Recovery of the Continuous Signal p. 496
- 9.5 How Do We Discretize the Derivative Operation? p. 500
- 9.6 Discretization of the State-Space Representation p. 504
- 9.7 The Bilinear Transformation and the Relationship between the Laplace-Domain and the z-Domain Representations p. 506
- 9.8 Other Transformation Methods p. 512
- 9.8.1 Impulse Invariance Method p. 512
- 9.8.2 The Step Invariance Method p. 512
- 9.8.3 The Forward Difference Method p. 512
- 9.8.4 The Backward Difference Method p. 512
- 9.8.5 The Bilinear Transformation p. 512
- 9.9 Some Insights p. 515
- 9.9.1 The Choice of the Sampling Interval T[subscript s] p. 515
- 9.9.2 The Effect of Choosing T[subscript s] on the Dynamics of the System p. 515
- 9.9.3 Does Sampling Introduce Additional Zeros for the Transfer Function H(z)? p. 516
- 9.10 End of Chapter Examples p. 517
- 9.11 End of Chapter Problems p. 534
- 10 Infinite Impulse Response (IIR) Filter Design p. 541
- 10.1 Introduction p. 541

- 10.2 The Design Process p. 542
- 10.2.1 Design Based on the Impulse Invariance Method p. 542
- 10.2.2 Design Based on the Bilinear Transform Method p. 545
- 10.3 IIR Filter Design Using Matlab p. 548
- 10.3.1 From the Analogue Prototype to the IIR Digital Filter p. 548
- 10.3.2 Direct Design p. 548
- 10.4 Some Insights p. 550
- 10.4.1 The Difficulty in Designing IIR Digital Filters in the z-Domain p. 550
- 10.4.2 Using the Impulse Invariance Method p. 552
- 10.4.3 The Choice of the Sampling Interval T[subscript s] p. 552
- 10.5 End of Chapter Examples p. 552
- 10.6 End of Chapter Problems p. 584
- 11 Finite Impulse Response (FIR) Digital Filters p. 591
- 11.1 Introduction p. 591
- 11.1.1 What Is an FIR Digital Filter? p. 591
- 11.1.2 A Motivating Example p. 591
- 11.2 FIR Filter Design p. 594
- 11.2.1 Stability of FIR Filters p. 596
- 11.2.2 Linear Phase of FIR Filters p. 597
- 11.3 Design Based on the Fourier Series: The Windowing Method p. 598
- 11.3.1 Ideal Lowpass FIR Filter Design p. 599
- 11.3.2 Other Ideal Digital FIR Filters p. 601
- 11.3.3 Windows Used in the Design of the Digital FIR Filter p. 602
- 11.3.4 Which Window Gives the Optimal h(n)? p. 604
- 11.3.5 Design of a Digital FIR Differentiator p. 605
- 11.3.6 Design of Comb FIR Filters p. 607
- 11.3.7 Design of a Digital Shifter: The Hilbert Transform Filter p. 609
- 11.4 From IIR to FIR Digital Filters: An Approximation p. 610
- 11.5 Frequency Sampling and FIR Filter Design p. 610
- 11.6 FIR Digital Design Using Matlab p. 611
- 11.6.1 Design Using Windows p. 611
- 11.6.2 Design Using Least-Squared Error p. 612
- 11.6.3 Design Using the Equiripple Linear Phase p. 612
- 11.6.4 How to Obtain the Frequency Response p. 612
- 11.7 Some Insights p. 613
- 11.7.1 Comparison with IIR Filters p. 613
- 11.7.2 The Different Methods Used in the FIR Filter Design p. 613
- 11.8 End of the Chapter Examples p. 614
- 11.9 End of Chapter Problems p. 644
- References p. 649
- Index p. 651