

- 1 Introduction p. 1
- 1.1 Overview of model-based predictive control p. 1
- 1.2 The main components of MPC p. 2
 - 1.2.1 Dependence of actions on predictions p. 2
 - 1.2.2 Predictions are based on a model p. 2
 - 1.2.3 Selecting the current input p. 3
 - 1.2.4 Receding horizon p. 4
 - 1.2.5 Optimal or safe performance p. 4
 - 1.2.6 Tuning p. 5
 - 1.2.7 Constraint handling p. 5
 - 1.2.8 Systematic use of future demands p. 6
 - 1.2.9 Systematic control design for multivariable systems p. 6
- 1.3 Overview of the book p. 7
- 1.4 Notation p. 9
 - 1.4.1 Table of notation p. 9
 - 1.4.2 Vectors of past and future values p. 9
 - 1.4.3 Toeplitz and Hankel matrices p. 9
 - 1.4.4 Common acronyms/abbreviations p. 14
- 2 Common linear models used in model predictive control p. 17
 - 2.1 Modelling uncertainty p. 18
 - 2.1.1 Integral action and disturbance models p. 18
 - 2.1.2 Modelling measurement noise p. 18
 - 2.2 Typical models p. 19
 - 2.3 State-space models p. 20
 - 2.3.1 Nominal state-space model p. 20
 - 2.3.2 Nonsquare systems p. 21
 - 2.3.3 Including a disturbance model p. 21
 - 2.3.4 Systematic inclusion of integral action with state-space models p. 22
 - 2.4.3 Achieving integral action with CARIMA models p. 25
 - 2.4 Transfer function models (single-input/single-output) p. 24
 - 2.4.1 Disturbance modelling p. 24
 - 2.4.2 Consistent steady-state estimates with CARIMA models p. 25
 - 2.4.4 Selection of $T(z)$ for MPC p. 26
 - 2.5 FIR models p. 26
 - 2.5.1 Impulse response models p. 26
 - 2.5.2 Step response models p. 27
 - 2.6 Independent models p. 27
 - 2.7 Matrix-fraction descriptions p. 28
 - 2.8 Modelling the dead times in a process p. 29
- 3 Prediction in model predictive control p. 31
 - 3.1 General format of prediction modelling p. 31
 - 3.2 Prediction with state-space models p. 32
 - 3.3.3 Prediction equations with $T(z)$ [not equal] 1 p. 36
 - 3.3 Prediction with transfer function models - matrix methods p. 33
 - 3.3.1 Prediction for a CARIMA model with $T(z) = 1$ - SISO case p. 33
 - 3.3.2 Prediction with CARIMA model and $T = 1$ - MIMO case p. 35

- 3.4 Using recursion to find matrices H, P, Q p. 39
- 3.5 Prediction with FIR models p. 41
 - 3.5.1 Impulse response models p. 41
 - 3.5.2 Step response models p. 42
- 3.6 Prediction with independent models p. 43
 - 3.6.1 Structure and prediction set up with internal models p. 43
 - 3.6.2 Prediction with unstable open-loop plant p. 44
- 3.7 Numerically robust prediction with open-loop unstable plant p. 48
 - 3.7.1 Why is open-loop prediction unsatisfactory? p. 48
 - 3.7.2 Prestabilisation and pseudo closed-loop prediction p. 49
- 3.8 Pseudo closed-loop prediction p. 52
- 4 Predictive control - the basic algorithm p. 53
 - 4.1 Summary of main results p. 53
 - 4.2 GPC algorithm - the main components p. 54
 - 4.2.1 Performance index and optimisation p. 54
 - 4.2.2 Restrictions on the predicted future control trajectory p. 56
 - 4.2.3 The receding horizon concept p. 57
 - 4.2.4 Constraints p. 57
 - 4.2.5 Multivariable systems p. 58
 - 4.2.6 The use of input increments and obtaining integral action p. 59
 - 4.2.7 Eliminating tracking offset while weighting the inputs p. 60
 - 4.2.8 Links to optimal control p. 61
 - 4.3 GPC algorithm formulation for transfer function models p. 61
 - 4.3.1 Steps to form a GPC control law p. 61
 - 4.3.2 Transfer function representation of the control law p. 62
 - 4.3.3 Closed-loop transfer functions p. 63
 - 4.3.4 GPC based on MFD models with a T-filter (GPCT) p. 64
 - 4.4 Predictive control with a state-space model p. 65
 - 4.4.1 Simple state augmentation p. 66
 - 4.4.2 State-space models without state augmentation p. 68
 - 4.5 Formulation for finite impulse response models p. 71
 - 4.6 Formulation for independent models p. 72
 - 4.6.1 IM is a transfer function or MFD p. 72
 - 4.6.2 Closed-loop poles in the IM case with an MFD model p. 73
 - 4.6.3 IM is a state-space model p. 73
 - 4.7 General comments on stability analysis of GPC p. 74
 - 4.8 Constraint handling p. 75
 - 4.8.1 The constraint equations p. 75
 - 4.8.2 Solving the constrained optimisation p. 78
 - 4.8.3 Hard and soft constraints p. 78
 - 4.8.4 Stability with constraints p. 79
 - 4.9 Simple variations on the basic algorithm p. 80
 - 4.9.1 Alternatives to the 2-norm p. 80
 - 4.9.2 Alternative parameterisations of the degrees of freedom p. 80
 - 4.9.3 Improving response to measurable disturbances p. 80
 - 4.10 Predictive functional control (PFC) p. 81

- 4.10.1 Predictive functional control with one coincidence point p. 81
- 4.10.2 PFC tuning parameters p. 82
- 4.10.3 PFC with two coincidence points p. 82
- 4.10.4 Limitations and summary p. 82
- 4.11 Other performance indices p. 83
- 5 Examples - tuning predictive control and numerical conditioning p. 85
- 5.2 Single-input/single-output examples p. 86
- 5.2.1 Effect of varying the output horizon p. 86
- 5.2.2 Effect of varying the input horizon p. 86
- 5.2.3 Effect of varying the control weighting p. 87
- 5.2.4 Summary p. 87
- 5.3 The benefits of systematic constraint handling p. 92
- 5.3.1 Simple example of weakness in a saturation policy p. 93
- 5.3.2 Numerical illustration of the weaknesses in saturation policies p. 93
- 5.4 Unstable examples p. 95
- 5.4.1 Effects of tuning parameters on unstable systems - a counter intuitive result p. 95
- 5.5 Numerical ill-conditioning with open-loop unstable systems p. 97
- 5.5.1 How does ill-conditioning arise? p. 98
- 5.6 MIMO examples p. 99
- 5.7 Summary of guidelines p. 101
- 6 Stability guarantees and optimising performance p. 103
- 6.1 Prediction mismatch in MPC p. 104
- 6.1.1 Illustration of ill-posed objective p. 104
- 6.1.2 Example where prediction mismatch causes instability p. 106
- 6.1.3 Summary p. 107
- 6.2 Feedforward design in MPC p. 107
- 6.2.1 Structure of the set point prefilter p. 108
- 6.2.2 Mismatch between predictions and actual behaviour p. 108
- 6.2.3 Making better use of advance information p. 109
- 6.3 Infinite horizons imply stability p. 112
- 6.3.1 Definition of the tail p. 113
- 6.3.2 Infinite horizons and the tail p. 114
- 6.3.3 Only the output horizon needs to be infinite p. 115
- 6.4 Stability proofs with constraints p. 116
- 6.4.1 Are infinite horizons impractical? p. 116
- 6.4.2 Alternatives to optimal control p. 117
- 6.5 Dual mode control - an overview p. 117
- 6.5.1 What is dual mode control? p. 117
- 6.6.1 The cost function for linear predictions over infinite horizons p. 121
- 5.1 Matching closed-loop and open-loop behaviour p. 85
- 6.5.2 The structure of dual mode predictions p. 118
- 6.5.3 Overview of MPC dual mode algorithms p. 118
- 6.5.4 Two possible dual mode algorithms p. 119
- 6.5.5 Is a dual mode strategy guaranteed stabilising? p. 119
- 6.5.6 How do dual mode predictions make infinite horizon MPC more tractable? p. 120
- 6.6 Implementation of dual mode MPC p. 121

- 6.6.2 Forming the cost function for dual mode predictions p. 122
- 6.6.3 Constraint handling with dual mode predictions p. 123
- 6.6.4 Computing the dual mode MPC control law p. 123
- 6.6.5 Remarks on stability and performance of dual mode control p. 124
- 7 Closed-loop paradigm p. 127
- 7.1 Introduction to the closed-loop paradigm p. 127
- 7.1.1 Overview of the CLP concept p. 128
- 7.1.2 CLP predictions p. 130
- 7.1.3 CLP structure p. 132
- 7.2 Setting up an MPC problem with the closed-loop paradigm p. 133
- 7.2.1 Setting up the cost function and computing the control law for state-space models p. 134
- 7.2.2 Including the constraints for state-space models p. 135
- 7.2.3 The constrained optimisation p. 136
- 7.3 Different choices for mode 2 of dual mode control p. 136
- 7.3.1 Dead beat terminal conditions (SGPC) p. 137
- 7.3.2 No terminal control (NTC) p. 140
- 7.3.3 Terminal mode by elimination of unstable modes (EUM) p. 140
- 7.3.4 Terminal mode is optimal (LQMP) p. 143
- 7.3.5 Summary of dual mode algorithms and key points p. 144
- 7.4 Are dual mode-based algorithms used in industry? p. 145
- 7.4.1 Efficacy of typical industrial algorithm p. 145
- 7.4.2 The potential role of dual mode algorithms p. 146
- 7.5 Advantages and disadvantages of the CLP over the open loop predictions p. 146
- 7.5.1 Cost function for optimal stable predictive control p. 147
- 7.5.2 Improved numerical conditioning with the CLP p. 149
- 7.5.3 Improved robust design with CLP p. 151
- 8 Constraint handling and feasibility issues in MPC p. 153
- 8.1 Introduction p. 153
- 8.1.1 Description of feasibility p. 153
- 8.1.2 Feasibility in MPC p. 154
- 8.1.3 Overview of chapter p. 154
- 8.2 Constraints in MPC p. 154
- 8.2.1 Hard constraints p. 155
- 8.2.2 Soft constraints p. 155
- 8.2.3 Terminal constraints p. 155
- 8.3 Why is feasibility important? p. 156
- 8.3.1 Consequences of infeasibility p. 156
- 8.3.2 Recursive feasibility p. 156
- 8.4 What causes infeasibility in predictive control? p. 158
- 8.4.1 Incompatible constraints due to overambitious performance requirements p. 158
- 8.4.2 Conflicts with terminal mode control laws p. 158
- 8.4.3 Model uncertainty p. 159
- 8.4.4 Unstable open-loop processes p. 159
- 8.5 Typical techniques for avoiding infeasibility p. 160
- 8.5.1 Constraint softening p. 160

- 8.6.1 Two reference governor algorithms p. 163
- 8.5.2 Back off and borders p. 161
- 8.5.3 Simple illustration of back off p. 162
- 8.6 Set point management and reference governor strategies p. 162
- 8.6.2 Links between the CLP and reference governor strategies p. 164
- 8.7 Non-dual mode algorithms and feasibility p. 164
- 8.8 Summary p. 165
- 9 Improving robustness--the constraint free case p. 167
- 9.1 Key concept used in robust design for MPC p. 167
- 9.2 Sensitivity functions for MPC with MFD models p. 168
- 9.2.1 Complementary sensitivity p. 168
- 9.2.2 Sensitivity functions used for robustness analysis p. 169
- 9.3 T-filter approach p. 171
- 9.3.1 Overview p. 171
- 9.3.2 How a T-filter is included p. 171
- 9.3.3 Illustration of the T-filter p. 173
- 9.4 Youla parameter approaches p. 174
- 9.4.1 Introducing a Youla parameterisation into a MPC control law p. 174
- 9.4.2 Effects of the Youla parameter on sensitivity p. 175
- 9.4.3 Optimising sensitivity p. 176
- 9.4.4 Use of Q on examples 1 and 2 p. 177
- 9.4.5 Example 3 p. 178
- 9.4.6 Potential improvements and comments p. 178
- 9.5 Internal model approaches p. 179
- 9.5.1 Nominal case p. 180
- 9.5.2 Uncertain case p. 180
- 10 The relationship between model structure and the robustness of MPC p. 185
- 10.1 Introduction p. 185
- 10.1.1 Importance of prediction errors p. 186
- 10.1.2 Overview of chapter p. 187
- 10.2 Summary of models to be compared p. 187
- 10.2.1 Prediction models and prediction errors p. 187
- 10.2.2 Analytical comparison of prediction errors from FIR, independent and transfer function models p. 190
- 10.3 Numerical examples of predictions errors p. 191
- 10.3.1 Details of simulation parameters p. 191
- 10.3.2 Summary of error comparisons p. 191
- 10.3.3 Conclusions p. 194
- 10.4 Effect of model structure on loop sensitivity p. 195
- 10.4.1 Control law structure and sensitivity functions p. 195
- 10.4.2 Numerical example p. 197
- 10.5 Conclusions p. 199
- 11 Robustness of MPC during constraint handling and invariant sets p. 205
- 11.1 Illustration of why robustness is hard to quantify during constraint handling p. 205
- 11.2 Feasibility p. 206
- 11.3 Simple methods for improving robustness during constraint handling p. 207

- 11.3.1 The T-filter p. 207
- 11.3.2 Youla approaches p. 208
- 11.4 Youla parameter and robust predictive control with constraint handling p. 210
- 11.4.1 Nominal feedback for use in the CLP p. 210
- 11.4.2 Introducing constraint handling p. 210
- 11.4.3 Constraint satisfaction for a set of plants p. 211
- 11.4.4 Simulation study p. 211
- 11.4.5 Conclusions p. 213
- 11.5 Using constraint tightening p. 213
- 11.6 Recursive feasibility for the uncertain case p. 215
- 11.7.3 Link between invariance and stability p. 218
- 11.7 Definition of invariant sets for unconstrained closed-loop systems p. 216
- 11.7.1 Ellipsoidal invariant sets p. 217
- 11.7.2 Polyhedral invariant sets p. 217
- 11.8 Invariance and constraint handling p. 219
- 11.8.1 Ensuring constraint satisfaction by set membership p. 219
- 11.8.2 Using invariant sets in predictive control p. 219
- 11.9 Computing invariant sets p. 221
- 11.9.1 Computing polyhedral invariant sets p. 221
- 11.9.2 Ellipsoidal sets p. 222
- 11.10 Invariance in the presence of uncertainty p. 224
- 11.10.1 Polyhedral sets and uncertainty p. 225
- 11.10.2 Ellipsoidal invariance in the presence of uncertainty p. 225
- 11.11 Using ellipsoidal invariant sets in robust MPC design p. 227
- 11.11.1 Overview p. 227
- 11.11.2 Algorithm of Kothare et al p. 227
- 11.11.3 Using the closed-loop paradigm p. 228
- 11.12 Conclusions p. 229
- 12 Optimisation and computational efficiency in predictive control p. 231
- 12.1 Optimisation algorithms in MPC p. 231
- 12.1.1 Active set methods p. 232
- 12.1.2 Interior point methods p. 233
- 12.1.3 Multi parametric quadratic programming (MPQP) p. 233
- 12.1.4 Simple but suboptimal approaches p. 234
- 12.2 Introduction to computationally efficient MPC p. 234
- 12.2.1 Concepts used to reduced on-line computation p. 235
- 12.3 Three computationally efficient algorithms using invariant sets p. 238
- 12.2.2 Invariant sets p. 235
- 12.2.3 Methods illustrated in this chapter p. 236
- 12.2.4 Notation and assumptions p. 237
- 12.3.1 Algorithms of [146], [156] (NESTED) p. 238
- 12.3.2 One d.o.f. algorithms (ONEDOF) p. 239
- 12.3.3 Algorithm of [65] p. 240
- 12.3.4 Overview of QPMPC, NESTED, ONEDOF, EMPC p. 242
- 12.3.5 Examples p. 242
- 12.3.6 Summary p. 248

- 12.4 Stability analysis and options within ONEDOF p. 248
- 12.4.1 Stability of ONEDOF using LMIs p. 248
- 12.4.2 Options in ONEDOF p. 249
- 12.5 Conclusions p. 251
- 13 Predictive functional control p. 253
- 13.1 Summary of the overall philosophy of PFC p. 253
- 13.2 Derivation of the PFC control law p. 254
- 13.2.1 Modelling and prediction p. 254
- 13.2.2 Desired reference trajectory p. 255
- 13.3 Tuning PFC p. 257
- 13.2.3 The coincidence points p. 255
- 13.2.4 Parameterisation of the d.o.f./future control trajectory p. 255
- 13.2.5 The control law p. 256
- 13.4 Constraint handling p. 258
- 13.5 Simulation examples with a single coincidence point p. 260
- 13.6 Unstable open-loop problems p. 260
- 13.6.1 Weaknesses of PFC when applied to unstable processes p. 261
- 13.6.2 Overcoming prediction mismatch by prediction stabilisation p. 262
- 13.6.3 Numerical examples p. 263
- 13.7 Conclusions p. 265
- 14 Multirate systems p. 271
- 14.1 An introduction to multirate systems p. 271
- 14.2 Background on model and controller structure p. 272
- 14.2.1 Modelling of MR systems p. 272
- 14.2.2 Control trajectory update p. 272
- 14.2.3 Control law structure in MR systems p. 273
- 14.2.4 Overview of chapter p. 275
- 14.3 GPC (finite output horizon) controllers for MR systems p. 276
- 14.3.1 The internal model/inferential control p. 276
- 14.3.2 MPC in the lifted environment p. 276
- 14.3.3 Comparison of IC and lifting based control schemes p. 278
- 14.4 Simulation example contrasting lifting and IC p. 279
- 14.4.1 Simulation details p. 279
- 14.4.2 Discussion of simulations p. 279
- 14.4.3 Summary of comparison p. 280
- 14.5 Infinite horizons in the MR environment p. 281
- 14.6 Conclusions p. 282
- 15 Modelling for predictive control p. 283
- 15.1 Introduction p. 283
- 15.1.1 Multi-models in MPC p. 283
- 15.1.2 Feasibility issues p. 284
- 15.1.3 Closed-loop identification and iterative feedback tuning p. 285
- 15.2 Predictions models p. 285
- 15.3 Identifying a multi-model p. 286
- 15.3.1 Algorithm details p. 287
- 15.3.2 Including a noise model in the multi-model p. 288

- 15.3.3 Over- and underparameterisation p. 288
- 15.4 Examples p. 289
- 15.4.1 No parameterisation errors p. 289
- 15.4.2 Examples with parameterisation errors p. 289
- 15.5 Conclusions p. 290
- 16 Conclusion p. 293
- A Appendix: Numerical examples and questions p. 295
- A.1 Numerical examples of prediction p. 295
- A.1.1 State-space predictions of eqn.(3.10) p. 295
- A.1.2 Prediction with transfer function models using eqn.(3.21) p. 296
- A.2 Numerical examples of control laws p. 297
- A.2.1 Control law of eqn. (4.53) p. 297
- A.2.2 Control law of eqn. (4.25) p. 298
- A.3 Typical questions for tutors p. 298
- A.3.1 Example tutorial questions p. 299
- A.3.2 Example exam questions p. 300
- B Appendix: References p. 303
- Index p. 315