

- Preface to the Second Edition p. v
- Preface to the First Edition p. vii
- 1. Equations of Heat Transfer and Fluid Mechanics p. 1
- 1.1 Introduction p. 1
- 1.1.1 Heat Transfer p. 1
- 1.1.2 Fluid Mechanics p. 2
- 1.2 Present Study p. 3
- 1.3 Governing Equations of a Continuum p. 3
- 1.3.1 Introduction p. 3
- 1.3.2 Conservation of Mass; the Continuity Equation p. 4
- 1.3.3 Conservation of Momenta p. 4
- 1.3.4 Conservation of Energy p. 5
- 1.3.5 Constitutive Equations p. 5
- 1.4 Governing Equations in Terms of Primitive Variables p. 7
- 1.4.1 Vector Form p. 7
- 1.4.2 Cartesian Component Form p. 7
- 1.4.3 Cylindrical Component Form p. 8
- 1.4.4 Closure p. 9
- 1.5 Porous Flow Equations p. 10
- 1.6 Auxiliary Transport Equations p. 11
- 1.7 Chemically Reacting Systems p. 12
- 1.8 Boundary Conditions p. 15
- 1.8.1 Viscous Flow Boundary Conditions p. 15
- 1.8.2 Porous Flow Boundary Conditions p. 18
- 1.8.3 Thermal and Transport Boundary Conditions p. 19
- 1.8.4 Initial Conditions p. 20
- 1.9 Change of Phase p. 21
- 1.10 Enclosure Radiation p. 23
- 1.11 Summary of Equations p. 25
- Problems p. 26
- References for Additional Reading p. 28
- 2. The Finite Element Method: An Overview p. 31
- 2.1 Introduction p. 31
- 2.2 Model Differential Equation p. 32
- 2.3 Finite Element Approximation p. 33
- 2.4 Weighted-Integral Statements and Weak Forms p. 35
- 2.5 Finite Element Model p. 38
- 2.6 Interpolation Functions p. 39
- 2.7 Assembly of Elements p. 43
- 2.8 Time-Dependent Problems p. 45
- 2.8.1 Introduction p. 45
- 2.8.2 Semidiscretization p. 46
- 2.8.3 Temporal Approximation p. 47
- 2.9 Axisymmetric Problems p. 48

- 2.10 Convective Boundary Conditions p. 51
- 2.11 Library of Finite Elements p. 52
- 2.11.1 Introduction p. 52
- 2.11.2 Triangular Elements p. 52
- 2.11.3 Rectangular Elements p. 54
- 2.12 Numerical Integration p. 55
- 2.12.1 Preliminary Comments p. 55
- 2.12.2 Coordinate Transformations p. 57
- 2.12.3 Integration Over a Master Rectangular Element p. 60
- 2.12.4 Integration Over a Master Triangular Element p. 61
- 2.13 Modeling Considerations p. 62
- 2.13.1 Mesh Generation p. 62
- 2.13.2 Representation of Boundary Flux p. 64
- 2.13.3 Imposition of Boundary Conditions p. 64
- 2.14 Illustrative Examples p. 65
- 2.14.1 Example 1 p. 66
- 2.14.2 Example 2 p. 72
- 2.14.3 Example 3 p. 73
- Problems p. 74
- References for Additional Reading p. 77
- 3. 3-D Conduction Heat Transfer p. 79
- 3.1 Introduction p. 79
- 3.2 Semidiscrete Finite Element Model p. 80
- 3.3 Interpolation Functions p. 83
- 3.3.1 Preliminary Comments p. 83
- 3.3.2 Hexahedral (Brick) Elements p. 83
- 3.3.3 Prism Elements p. 85
- 3.3.4 Tetrahedral Elements p. 86
- 3.4 Numerical Integration p. 87
- 3.5 Computation of Surface Flux p. 88
- 3.6 Semidiscrete Finite Element Model p. 91
- 3.7 Solution of Nonlinear Equations p. 92
- 3.7.1 Preliminary Comments p. 92
- 3.7.2 Steady-State Problems p. 92
- 3.7.3 Transient Problems p. 94
- 3.8 Radiation Solution Algorithms p. 104
- 3.9 Variable Properties p. 108
- 3.9.1 Temperature-Dependent Properties p. 108
- 3.9.2 Phase Change Properties p. 109
- 3.9.3 Anisotropic Properties p. 111
- 3.10 Post-Processing Operations p. 112
- 3.10.1 Heat Flux p. 112
- 3.10.2 Heat Flow Function p. 114
- 3.11 Advanced Topics in Conduction p. 115

- 3.11.1 Introduction p. 115
- 3.11.2 Specialty Elements p. 116
- 3.11.3 Computational Boundary Conditions p. 119
- 3.11.4 Bulk Nodes p. 125
- 3.11.5 Reactive Materials p. 127
- 3.11.6 Material Motion p. 129
- 3.12 Example Problems p. 130
- 3.12.1 Introduction p. 130
- 3.12.2 Temperature-Dependent Conductivity p. 131
- 3.12.3 Anisotropic Conductivity p. 131
- 3.12.4 One-Dimensional Stefan Problem p. 133
- 3.12.5 Drag Bit Analysis p. 135
- 3.12.6 Brazing and Welding Analyses p. 136
- 3.12.7 Investment Casting p. 141
- Problems p. 143
- References for Additional Reading p. 144
- 4. Viscous Incompressible Flows p. 149
- 4.1 Introduction p. 149
- 4.1.1 Background p. 149
- 4.1.2 Governing Equations p. 149
- 4.2 Mixed Finite Element Model p. 152
- 4.2.1 Weak Form p. 152
- 4.2.2 Finite Element Model p. 153
- 4.3 Penalty Finite Element Models p. 156
- 4.3.1 Introduction p. 156
- 4.3.2 Penalty Function Method p. 157
- 4.3.3 Reduced Integration Penalty Model p. 159
- 4.3.4 Consistent Penalty Model p. 160
- 4.4 Finite Element Models of Porous Flow p. 161
- 4.5 Computational Considerations p. 163
- 4.5.1 Properties of the Matrix Equations p. 163
- 4.5.2 Choice of Interpolation Functions p. 164
- 4.5.3 Evaluation of Element Matrices in Penalty Models p. 169
- 4.5.4 Pressure Calculation p. 170
- 4.5.5 Traction Boundary Conditions p. 172
- 4.6 Solution of Nonlinear Equations p. 175
- 4.6.1 General Discussion p. 175
- 4.6.2 Fully Coupled Solution Methods p. 178
- 4.6.3 Pressure Correction/Projection Methods p. 183
- 4.7 Time-Approximation Schemes p. 186
- 4.7.1 Preliminary Comments p. 186
- 4.7.2 Forward/Backward Euler Schemes p. 186
- 4.7.3 Adams-Bashforth/Trapezoid Rule p. 187
- 4.7.4 Implicit Integration and Time Step Control p. 188

- 4.7.5 Explicit Integration p. 189
- 4.8 Stabilized Methods p. 190
- 4.8.1 Preliminary Comments p. 190
- 4.8.2 Galerkin/Least-Squares Formulation p. 191
- 4.9 Post-Processing p. 194
- 4.9.1 Stress Computation p. 194
- 4.9.2 Stream Function Computation p. 196
- 4.9.3 Particle Tracking p. 198
- 4.10 Advanced Topics - Free Surface Flows p. 198
- 4.10.1 Preliminary Comments p. 198
- 4.10.2 Time-Independent Free Surfaces p. 199
- 4.10.3 Time-Dependent Free Surfaces p. 204
- 4.11 Advanced Topics - Turbulence p. 211
- 4.11.1 Preliminary Comments p. 211
- 4.11.2 Governing Equations p. 212
- 4.11.3 General Turbulence Models p. 213
- 4.11.4 One-Point Closure Turbulence Models p. 215
- 4.11.5 Finite Element Modeling of Turbulence p. 219
- 4.12 Numerical Examples p. 221
- 4.12.1 Preliminary Comments p. 221
- 4.12.2 Fluid Squeezed between Parallel Plates p. 222
- 4.12.3 Flow of a Viscous Lubricant in a Slider Bearing p. 225
- 4.12.4 Wall-Driven 2-D Cavity Flow p. 226
- 4.12.5 Wall-Driven 3-D Cavity Flow p. 229
- 4.12.6 Evaluation of the EBE Iterative Solvers p. 229
- 4.12.7 Backward Facing Step p. 233
- 4.12.8 Flow Past a Submarine p. 235
- 4.12.9 Crystal Growth from the Melt p. 237
- 4.12.10 Mold Filling p. 238
- Problems p. 242
- References for Additional Reading p. 243
- 5. Convective Heat Transfer p. 255
- 5.1 Introduction p. 255
- 5.1.1 Background p. 255
- 5.1.2 Governing Equations p. 255
- 5.2 Mixed Finite Element Model p. 257
- 5.3 Penalty Finite Element Model p. 261
- 5.3.1 Preliminary Comments p. 261
- 5.3.2 Reduced Integration Penalty Model p. 262
- 5.3.3 Consistent Penalty Model p. 263
- 5.4 Finite Element Models of Porous Flow p. 263
- 5.5 Solution Methods p. 265
- 5.5.1 General Discussion p. 265
- 5.5.2 Newton's Method p. 266

- 5.5.3 Segregated Equation Methods p. 267
- 5.6 Convection with Change of Phase p. 269
- 5.7 Convection with Enclosure Radiation p. 271
- 5.8 Post-Computation of Heat Flux p. 271
- 5.9 Advanced Topics - Turbulent Heat Transfer p. 273
- 5.10 Advanced Topics - Chemically Reacting Systems p. 274
- 5.10.1 Preliminary Comments p. 274
- 5.10.2 Finite Element Modeling of Chemical Reactions p. 274
- 5.11 Numerical Examples p. 275
- 5.11.1 Preliminary Comments p. 275
- 5.11.2 Concentric Tube Flow p. 275
- 5.11.3 Tube Flow with Change of Phase p. 276
- 5.11.4 Heated Cavity p. 278
- 5.11.5 Solar Receiver p. 279
- 5.11.6 Tube Bundle p. 282
- 5.11.7 Volumetrically Heated Fluid p. 284
- 5.11.8 Porous/Fluid Layer p. 287
- 5.11.9 Curing of An Epoxy p. 289
- References for Additional Reading p. 292
- 6. Non-Newtonian Fluids p. 295
- 6.1 Introduction p. 295
- 6.2 Governing Equations of Inelastic Fluids p. 296
- 6.2.1 Conservation Equations p. 296
- 6.2.2 Boundary Conditions p. 297
- 6.2.3 Constitutive Equations p. 298
- 6.3 Finite Element Models of Inelastic Fluids p. 301
- 6.3.1 Introduction p. 301
- 6.3.2 Mixed Model p. 302
- 6.3.3 Penalty Model p. 304
- 6.3.4 Matrix Evaluations p. 305
- 6.4 Solution Methods for Inelastic Fluids p. 307
- 6.5 Governing Equations of Viscoelastic Fluids p. 311
- 6.5.1 Conservation Equations p. 311
- 6.5.2 Constitutive Equations p. 312
- 6.5.3 Boundary Conditions p. 318
- 6.6 Finite Element Model of Differential Form p. 318
- 6.6.1 Preliminary Comments p. 318
- 6.6.2 Summary of Governing Equations p. 318
- 6.6.3 Finite Element Model p. 320
- 6.6.4 Solution Methods p. 324
- 6.7 Additional Models of Differential Form p. 325
- 6.7.1 Explicitly Elliptic Momentum Equation Method p. 326
- 6.7.2 Elastic Viscous Stress Splitting Method p. 327
- 6.8 Finite Element Model of Integral Form p. 329

- 6.9 Unresolved Problems p. 331
- 6.9.1 General Comments p. 331
- 6.9.2 Choice of Constitutive Equation p. 332
- 6.9.3 Solution Uniqueness and Existence p. 332
- 6.9.4 Numerical Algorithm Problems p. 333
- 6.9.5 Equation Change of Type p. 334
- 6.9.6 Closure p. 335
- 6.10 Numerical Examples p. 335
- 6.10.1 Preliminary Comments p. 335
- 6.10.2 Buoyancy Driven Flow in a Cavity p. 336
- 6.10.3 Driven Cavity Flow p. 336
- 6.10.4 Squeeze Film Flow p. 339
- 6.10.5 Time-Dependent Poiseuille Flow p. 341
- 6.10.6 Four-to-One Contraction Problem p. 343
- Problems p. 346
- References for Additional Reading p. 346
- 7. Coupled Problems p. 353
- 7.1 Introduction p. 353
- 7.2 Coupled Boundary Value Problems p. 353
- 7.3 Fluid Mechanics and Heat Transfer p. 354
- 7.3.1 Introduction p. 354
- 7.3.2 Continuum Equations p. 355
- 7.3.3 Finite Element Models p. 356
- 7.4 Solid Mechanics p. 357
- 7.4.1 Introduction p. 357
- 7.4.2 Continuum Equations p. 357
- 7.4.3 Constitutive Relations p. 360
- 7.4.4 Boundary Conditions p. 361
- 7.4.5 Finite Element Models p. 361
- 7.4.6 Solution Methods - Quasi-Static Solid Mechanics p. 363
- 7.5 Electromagnetics p. 363
- 7.5.1 Introduction p. 363
- 7.5.2 Maxwell's Equations p. 364
- 7.5.3 Electromagnetic Potentials p. 367
- 7.5.4 Boundary and Interface Conditions p. 370
- 7.5.5 Gauge Conditions p. 372
- 7.5.6 Static Field Problems p. 372
- 7.5.7 Finite Element Models for EM Fields p. 374
- 7.5.8 Solution Methods - EM Fields p. 379
- 7.6 Coupled Problems in Mechanics p. 380
- 7.6.1 Introduction p. 380
- 7.6.2 Heat Conduction - Viscous Fluid Interactions 1 and 2 p. 381
- 7.6.3 Heat Conduction - Quasi-Static Interactions 1 and 3 p. 381
- 7.6.4 Heat Conduction - Electric Field Interactions 1 and 4 p. 383

- 7.6.5 Heat Conduction - Electromagnetic Field Interactions 1 and 4 and 5 p. 384
- 7.6.6 Viscous Flow - Quasi-Static Solid Interactions 2 and 3 p. 386
- 7.6.7 Viscous Flow - Electric Field Interactions 2 and 4 p. 387
- 7.6.8 Viscous Flow - Electromagnetic Field Interactions 2 and 4 and 5 p. 388
- 7.6.9 Quasi-Static Solid - Electromagnetic Field Interactions 3 and 4 and 5 p. 389
- 7.7 Implementation of Coupled Algorithms p. 390
- 7.8 Numerical Examples p. 392
  - 7.8.1 Introduction p. 392
  - 7.8.2 Thermal-Stress Example p. 392
  - 7.8.3 Thermal-Electromagnetic Example p. 395
  - 7.8.4 Fluid-Solid Interaction Example p. 398
  - 7.8.5 Fluid-Electromagnetic Example p. 400
  - References for Additional Reading p. 402
- 8. Advanced Topics: Parallel Processing p. 405
  - 8.1 Introduction p. 405
  - 8.2 Parallel Systems p. 406
    - 8.2.1 Classification p. 406
    - 8.2.2 Languages and Communication Utilities p. 408
    - 8.2.3 Performance p. 409
  - 8.3 FEM and Parallel Processing p. 411
    - 8.3.1 Preliminary Comments p. 411
    - 8.3.2 Generic FEM Steps p. 411
    - 8.3.3 External Preprocessing p. 412
    - 8.3.4 Internal Preprocessing p. 414
    - 8.3.5 Solution Processing p. 414
    - 8.3.6 Internal Postprocessing p. 418
    - 8.3.7 External Postprocessing p. 419
    - 8.3.8 Other Parallel Issues p. 419
  - 8.4 Summary p. 421
    - References for Additional Reading p. 421
- Appendix A Computer Program FEM2DHT p. 425
  - A.1 Introduction p. 425
  - A.2 Heat Transfer and Related Problems p. 425
  - A.3 Flows of Viscous Incompressible Fluids p. 426
  - A.4 Description of the Input Data p. 426
  - A.5 Source Listings of Selective Subroutines p. 436
  - Reference p. 436
- Appendix B Solution of Linear Equations p. 443
  - B.1 Introduction p. 443
  - B.2 Direct Methods p. 444
  - B.3 Iterative Methods p. 445
    - B.3.1 General Comments p. 445
    - B.3.2 Solution Algorithms p. 446
  - References for Additional Reading p. 450

- Appendix C Fixed Point Methods and Contraction Mappings p. 455
- C.1 Fixed Point Theorem p. 455
- C.2 Chord Method p. 457
- C.3 Newton's Method p. 457
- C.4 The Newton-Raphson Method p. 458
- C.5 Descent Methods p. 459
- References for Additional Reading p. 459
- Subject Index p. 461