- Authors p. xxi
- Foreword p. xxiii
- Notations and Some Remarks p. xxv
- Introduction: Some Definitions, Formulas, Methods, and Transformations p. 1
- 0.1. First-Order Differential Equations p. 1
- 0.1.1. General Concepts. The Cauchy Problem. Uniqueness and Existence Theorems p. 1
- 0.1.2. Equations Solved for the Derivative. Simplest Techniques of Integration p. 3
- 0.1.3. Exact Differential Equations. Integrating Factor p. 5
- 0.1.4. Riccati Equation p. 6
- 0.1.5. Abel Equations of the First Kind p. 9
- 0.1.6. Abel Equations of the Second Kind p. 10
- 0.1.7. Equations Not Solved for the Derivative p. 14
- 0.1.8. Contact Transformations p. 15
- 0.1.9. Approximate Analytic Methods for Solution of Equations p. 18
- 0.1.10. Numerical Integration of Differential Equations p. 20
- 0.2. Second-Order Linear Differential Equations p. 21
- 0.2.1. Formulas for the General Solution. Some Transformations p. 21
- 0.2.2. Representation of Solutions as a Series in the Independent Variable p. 23
- 0.2.3. Asymptotic Solutions p. 24
- 0.2.4. Boundary Value Problems p. 27
- 0.2.5. Eigenvalue Problems p. 29
- 0.3. Second-Order Nonlinear Differential Equations p. 33
- 0.3.1. Form of the General Solution. Cauchy Problem p. 33
- 0.3.2. Equations Admitting Reduction of Order p. 34
- 0.3.3. Methods of Regular Series Expansions with Respect to the Independent Variable or Small Parameter p. 37
- 0.3.4. Perturbation Methods of Mechanics and Physics p. 40
- 0.3.5. Galerkin Method and Its Modifications (Projection Methods) p. 46
- 0.3.6. Iteration and Numerical Methods p. 49
- 0.4. Linear Equations of Arbitrary Order p. 51
- 0.4.1. Linear Equations with Constant Coefficients p. 51
- 0.4.2. Linear Equations with Variable Coefficients p. 52
- 0.4.3. Asymptotic Solutions of Linear Equations p. 54
- 0.5. Nonlinear Equations of Arbitrary Order p. 56
- 0.5.1. Structure of the General Solution. Cauchy Problem p. 56
- 0.5.2. Equations Admitting Reduction of Order p. 56
- 0.5.3. A Method for Construction of Solvable Equations of General Form p. 59
- 0.6. Lie Group and Discrete-Group Methods p. 60
- 0.6.1. Lie Group Method. Point Transformations p. 60
- 0.6.2. Contact Transformations. Backlund Transformations. Formal Operators. Factorization Principle p. 65
- 0.6.3. First Integrals (Conservation Laws) p. 71
- 0.6.4. Discrete-Group Method. Point Transformations p. 73
- 0.6.5. Discrete-Group Method. The Method of RF-Pairs p. 75
- 1. First-Order Differential Equations p. 81

- 1.1. Simplest Equations with Arbitrary Functions Integrable in Closed Form p. 81
- 1.1.1. Equations of the Form y'[subscript x] = f(x) p. 81
- 1.1.2. Equations of the Form y'[subscript x] = f(y) p. 81
- 1.1.3. Separable Equations y'[subscript x] = f(x)g(y) p. 81
- 1.1.4. Linear Equation g(x)y'[subscript x] = f[subscript 1](x)y + f[subscript 0](x) p. 81
- 1.1.5. Bernoulli Equation g(x)y'[subscript x] = f[subscript 1](x)y + f[subscript n](x)y[superscript n] p. 81
- 1.1.6. Homogeneous Equation y'[subscript x] = f(y/x) p. 82
- 1.2. Riccati Equation g(x)y'[subscript x] = f[subscript 2](x)y[superscript 2] + f[subscript 1](x)y + f[subscript 0](x) p. 82
- 1.2.1. Preliminary Remarks p. 82
- 1.2.2. Equations Containing Power Functions p. 82
- 1.2.3. Equations Containing Exponential Functions p. 89
- 1.2.4. Equations Containing Hyperbolic Functions p. 92
- 1.2.5. Equations Containing Logarithmic Functions p. 94
- 1.2.6. Equations Containing Trigonometric Functions p. 95
- 1.2.7. Equations Containing Inverse Trigonometric Functions p. 100
- 1.2.8. Equations with Arbitrary Functions p. 102
- 1.2.9. Some Transformations p. 105
- 1.3. Abel Equations of the Second Kind p. 107
- 1.3.1. Equations of the Form yy'[subscript x] - y = f(x) p. 107
- 1.3.2. Equations of the Form yy'[subscript x] = f(x)y + 1 p. 120
- 1.3.3. Equations of the Form yy'[subscript x] = f[subscript 1](x)y + f[subscript 0](x) p. 121
- 1.3.4. Equations of the Form [g[subscript 1](x)y + g[subscript 0](x)]y'[subscript x] = f[subscript 2](x)y[superscript 2] + f[subscript 1](x)y + f[subscript 0](x) p. 132
- 1.3.5. Some Types of First- and Second-Order Equations Reducible to Abel Equations of the Second Kind p. 136
- 1.4. Equations Containing Polynomial Functions of y p. 138
- 1.4.1. Abel Equations of the First Kind y'[subscript x] = f[subscript 3](x)y[superscript 3] + f[subscript 2](x)y[superscript 2] + f[subscript 1](x)y + f[subscript 0](x) p. 138
- 1.4.2. Equations of the Form (A[subscript 22]y[superscript 2] + A[subscript 12]xy + A[subscript 11]x[superscript 2] + A[subscript 0])y'[subscript x] = B[subscript 22]y[superscript 2] + B[subscript 12]xy + B[subscript 11]x[superscript 2] + B[subscript 0] p. 142
- 1.4.3. Equations of the Form (A[subscript 22]y[superscript 2] + A[subscript 12]xy + A[subscript 11]x[superscript 2] + A[subscript 2]y + A[subscript 1]x)y'[subscript x] = B[subscript 22]y[superscript 2] + B[subscript 12]xy + B[subscript 11]x[superscript 2] + B[subscript 2]y + B[subscript 1]x p. 144
- 1.4.4. Equations of the Form (A[subscript 22]y[superscript 2] + A[subscript 12]xy + A[subscript 11]x[superscript 2] + A[subscript 2]y + A[subscript 1]x + A[subscript 0])y'[subscript x] = B[subscript 22]y[superscript 2] + B[subscript 12]xy + B[subscript 11]x[superscript 2] + B[subscript 2]y + B[subscript 1]x + B[subscript 0] p. 151
- 1.4.5. Equations of the Form (A[subscript 3]y[superscript 3] + A[subscript 2]xy[superscript 2] + A[subscript 1]x[superscript 2]y + A[subscript 0]x[superscript 3] + a[subscript 1]y + a[subscript 0]x)y'[subscript x] = B[subscript 3]y[superscript 3] +

B[subscript 2]xy[superscript 2] + B[subscript 1]x[superscript 2]y + B[subscript 0]x[superscript 3] + b[subscript 1]y + b[subscript 0]x p. 155
- 1.5. Equations of the Form f(x, y)y'[subscript x] = g(x, y) Containing Arbitrary Parameters p. 159
- 1.5.1. Equations Containing Power Functions p. 159
- 1.5.2. Equations Containing Exponential Functions p. 162
- 1.5.3. Equations Containing Hyperbolic Functions p. 166
- 1.5.4. Equations Containing Logarithmic Functions p. 168
- 1.5.5. Equations Containing Trigonometric Functions p. 169
- 1.5.6. Equations Containing Combinations of Exponential, Hyperbolic, Logarithmic, and Trigonometric Functions p. 171
- 1.6. Equations of the Form F(x, y, y'[subscript x]) = 0 Containing Arbitrary Parameters p. 173
- 1.6.1. Equations of the Second Degree in y'[subscript x] p. 173
- 1.6.2. Equations of the Third Degree in y'[subscript x] p. 180
- 1.6.3. Equations of the Form (y'[subscript x])[superscript k] = f(y) + g(x) p. 182
- 1.6.4. Other Equations p. 190
- 1.7. Equations of the Form f(x, y)y'[subscript x] = g(x, y) Containing Arbitrary Functions p. 195
- 1.7.1. Equations Containing Power Functions p. 195
- 1.7.2. Equations Containing Exponential and Hyperbolic Functions p. 197
- 1.7.3. Equations Containing Logarithmic Functions p. 199
- 1.7.4. Equations Containing Trigonometric Functions p. 200
- 1.7.5. Equations Containing Combinations of Exponential, Logarithmic, and Trigonometric Functions p. 201
- 1.8. Equations of the Form F(x, y, y'[subscript x]) = 0 Containing Arbitrary Functions p. 203
- 1.8.1. Some Equations p. 203
- 1.8.2. Some Transformations p. 212
- 2. Second-Order Differential Equations p. 213
- 2.1. Linear Equations p. 213
- 2.1.1. Representation of the General Solution Through a Particular Solution p. 213
- 2.1.2. Equations Containing Power Functions p. 213
- 2.1.3. Equations Containing Exponential Functions p. 246
- 2.1.4. Equations Containing Hyperbolic Functions p. 252
- 2.1.5. Equations Containing Logarithmic Functions p. 257
- 2.1.6. Equations Containing Trigonometric Functions p. 260
- 2.1.7. Equations Containing Inverse Trigonometric Functions p. 271
- 2.1.8. Equations Containing Combinations of Exponential, Logarithmic, Trigonometric, and Other Functions p. 277
- 2.1.9. Equations with Arbitrary Functions p. 285
- 2.1.10. Some Transformations p. 292
- 2.2. Autonomous Equations y''[subscript x x] = F(y, y'[subscript x]) p. 295
- 2.2.1. Equations of the Form y''[subscript x x] - y'[subscript x] = f(y) p. 295
- 2.2.2. Equations of the Form y''[subscript x x] + f(y)y'[subscript x] + y = 0 p. 299
- 2.2.3. Lienard Equations y''[subscript x x] + f(y)y'[subscript x] + g(y) = 0 p. 301

- 2.2.4. Rayleigh Equations y''[subscript x x] + f(y'[subscript x]) + g(y) = 0  p. 304
- 2.3. Emden-Fowler Equation y''[subscript x x] = Ax[superscript n]y[superscript m]  p. 306
- 2.3.1. Exact Solutions  p. 306
- 2.3.2. First Integrals (Conservation Laws)  p. 312
- 2.3.3. Some Formulas and Transformations  p. 314
- 2.4. Equations of the Form y''[subscript x x] = A[subscript 1]x[superscript n[subscript 1]y[superscript m[subscript 1] + A[subscript 2]x[superscript n[subscript 2]y[superscript m[subscript 2]  p. 314
- 2.4.1. Classification Table  p. 314
- 2.4.2. Exact Solutions  p. 318
- 2.5. Generalized Emden-Fowler Equation y''[subscript x x] = Ax[superscript n]y[superscript m](y'[subscript x])[superscript l]  p. 336
- 2.5.1. Classification Table  p. 336
- 2.5.2. Exact Solutions  p. 339
- 2.5.3. Some Formulas and Transformations  p. 354
- 2.6. Equations of the Form y''[subscript x x] = A[subscript 1]x[superscript n[subscript 1]y[superscript m[subscript 1](y'[subscript x])[superscript l[subscript 1] + A[subscript 2]x[superscript n[subscript 2]y[superscript m[subscript 2](y'[subscript x])[superscript l[subscript 2]  p. 356
- 2.6.1. Modified Emden-Fowler Equation y''[subscript x x] = A[subscript 1]x[superscript -1]y'[subscript x] + A[subscript 2]x[superscript n]y[superscript m]  p. 356
- 2.6.2. Equations of the Form y''[subscript x x] = (A[subscript 1]x[superscript n[subscript 1]y[superscript m[subscript 1] + A[subscript 2]x[superscript n[subscript 2]y[superscript m[subscript 2])(y'[subscript x])[superscript l]  p. 365
- 2.6.3. Equations of the Form y''[subscript x x] = [sigma] Ax[superscript n]y[superscript m](y'[subscript x])[superscript l] + Ax[superscript n-1]y[superscript m+1](y'[subscript x])[superscript l-1]  p. 393
- 2.6.4. Other Equations (l[subscript 1] [not equal] l[subscript 2])  p. 406
- 2.7. Equations of the Form y''[subscript x x] = f(x)g(y)h(y'[subscript x])  p. 411
- 2.7.1. Equations of the Form y''[subscript x x] = f(x)g(y)  p. 412
- 2.7.2. Equations Containing Power Functions (h [characters not reproducible] const)  p. 414
- 2.7.3. Equations Containing Exponential Functions (h [characters not reproducible] const)  p. 418
- 2.7.4. Equations Containing Hyperbolic Functions (h [characters not reproducible] const)  p. 421
- 2.7.5. Equations Containing Trigonometric Functions (h [characters not reproducible] const)  p. 423
- 2.7.6. Some Transformations  p. 424
- 2.8. Some Nonlinear Equations with Arbitrary Parameters  p. 425
- 2.8.1. Equations Containing Power Functions  p. 425
- 2.8.2. Painleve Transcendents  p. 432
- 2.8.3. Equations Containing Exponential Functions  p. 438
- 2.8.4. Equations Containing Hyperbolic Functions  p. 445
- 2.8.5. Equations Containing Logarithmic Functions  p. 450

- 2.8.6. Equations Containing Trigonometric Functions p. 452
- 2.8.7. Equations Containing the Combinations of Exponential, Hyperbolic, Logarithmic, and Trigonometric Functions p. 459
- 2.9. Equations Containing Arbitrary Functions p. 461
- 2.9.1. Equations of the Form F(x, y)y''[subscript xx] + G(x, y) = 0 p. 461
- 2.9.2. Equations of the Form F(x, y)y''[subscript xx] + G(x, y)y'[subscript x] + H(x, y) = 0 p. 467
- 2.9.3. Equations of the Form F(x, y)y''[subscript xx] + [characters not reproducible] G[subscript m](x, y)(y'[subscript x])[superscript m] = 0 (M = 2, 3, 4) p. 471
- 2.9.4. Equations of the Form F(x, y, y'[subscript x])y''[subscript xx] + G(x, y, y'[subscript x]) = 0 p. 475
- 2.9.5. Equations Not Solved for Second Derivative p. 484
- 2.9.6. Equations of General Form p. 486
- 2.9.7. Some Transformations p. 492
- 3. Third-Order Differential Equations p. 495
- 3.1. Linear Equations p. 495
- 3.1.1. Preliminary Remarks p. 495
- 3.1.2. Equations Containing Power Functions p. 496
- 3.1.3. Equations Containing Exponential Functions p. 512
- 3.1.4. Equations Containing Hyperbolic Functions p. 516
- 3.1.5. Equations Containing Logarithmic Functions p. 525
- 3.1.6. Equations Containing Trigonometric Functions p. 528
- 3.1.7. Equations Containing Inverse Trigonometric Functions p. 539
- 3.1.8. Equations Containing Combinations of Exponential, Logarithmic, Trigonometric, and Other Functions p. 544
- 3.1.9. Equations Containing Arbitrary Functions p. 550
- 3.2. Equations of the Form y'''[subscript xxx] = Ax[superscript [alpha]y[superscript [beta](y'[subscript x])[superscript gamma](y''[subscript xx])[superscript [delta ] p. 559
- 3.2.1. Classification Table p. 559
- 3.2.2. Equations of the Form y'''[subscript xxx] = Ay[superscript beta] p. 566
- 3.2.3. Equations of the Form y'''[subscript xxx] = Ax[superscript alpha]y[superscript beta] p. 567
- 3.2.4. Equations with |[gamma]| + |[delta]| [not equal] 0 p. 568
- 3.2.5. Some Transformations p. 592
- 3.3. Equations of the Form y'''[subscript xxx] = f(y)g(y'[subscript x])h(y''[subscript xx]) p. 592
- 3.3.1. Equations Containing Power Functions p. 592
- 3.3.2. Equations Containing Exponential Functions p. 595
- 3.3.3. Other Equations p. 599
- 3.4. Nonlinear Equations with Arbitrary Parameters p. 601
- 3.4.1. Equations Containing Power Functions p. 601
- 3.4.2. Equations Containing Exponential Functions p. 608
- 3.4.3. Equations Containing Hyperbolic Functions p. 611
- 3.4.4. Equations Containing Logarithmic Functions p. 616
- 3.4.5. Equations Containing Trigonometric Functions p. 618
- 3.5. Nonlinear Equations Containing Arbitrary Functions p. 622

- 3.5.1. Equations of the Form F(x, y)y'"[subscript xxx] + G(x, y) = 0 p. 622
- 3.5.2. Equations of the Form F(x, y, y'[subscript x])y'"[subscript xxx] + G(x, y, y'[subscript x]) = 0 p. 624
- 3.5.3. Equations of the Form F(x, y, y'[subscript x])y'"[subscript xxx] + G(x, y, y'[subscript x])y"[subscript xx] + H(x, y, y'[subscript x]) = 0 p. 629
- 3.5.4. Equations of the Form F(x, y, y'[subscript x])y'"[subscript xxx] + [characters not reproducible]G[subscript alpha](x, y, y'[subscript x])(y"[subscript xx])[superscript alpha] = 0 p. 634
- 3.5.5. Other Equations p. 636
- 4. Fourth-Order Differential Equations p. 641
- 4.1. Linear Equations p. 641
- 4.1.1. Preliminary Remarks p. 641
- 4.1.2. Equations Containing Power Functions p. 641
- 4.1.3. Equations Containing Exponential and Hyperbolic Functions p. 648
- 4.1.4. Equations Containing Logarithmic Functions p. 651
- 4.1.5. Equations Containing Trigonometric Functions p. 652
- 4.1.6. Equations Containing Arbitrary Functions p. 655
- 4.2. Nonlinear Equations p. 659
- 4.2.1. Equations Containing Power Functions p. 659
- 4.2.2. Equations Containing Exponential Functions p. 666
- 4.2.3. Equations Containing Hyperbolic Functions p. 668
- 4.2.4. Equations Containing Logarithmic Functions p. 672
- 4.2.5. Equations Containing Trigonometric Functions p. 674
- 4.2.6. Equations Containing Arbitrary Functions p. 678
- 5. Higher-Order Differential Equations p. 689
- 5.1. Linear Equations p. 689
- 5.1.1. Preliminary Remarks p. 689
- 5.1.2. Equations Containing Power Functions p. 689
- 5.1.3. Equations Containing Exponential and Hyperbolic Functions p. 695
- 5.1.4. Equations Containing Logarithmic Functions p. 698
- 5.1.5. Equations Containing Trigonometric Functions p. 698
- 5.1.6. Equations Containing Arbitrary Functions p. 701
- 5.2. Nonlinear Equations p. 705
- 5.2.1. Equations Containing Power Functions p. 705
- 5.2.2. Equations Containing Exponential Functions p. 711
- 5.2.3. Equations Containing Hyperbolic Functions p. 713
- 5.2.4. Equations Containing Logarithmic Functions p. 717
- 5.2.5. Equations Containing Trigonometric Functions p. 718
- 5.2.6. Equations Containing Arbitrary Functions p. 722
- Supplements p. 735
- S.1. Elementary Functions and Their Properties p. 735
- S.1.1. Trigonometric Functions p. 735
- S.1.2. Hyperbolic Functions p. 738
- S.1.3. Inverse Trigonometric Functions p. 740
- S.1.4. Inverse Hyperbolic Functions p. 742
- S.2. Special Functions and Their Properties p. 743

- S.2.1. Some Symbols and Coefficients p. 743
- S.2.2. Error Functions and Exponential Integral p. 744
- S.2.3. Gamma and Beta Functions p. 745
- S.2.4. Incomplete Gamma and Beta Functions p. 747
- S.2.5. Bessel Functions p. 748
- S.2.6. Modified Bessel Functions p. 751
- S.2.7. Degenerate Hypergeometric Functions p. 753
- S.2.8. Hypergeometric Functions p. 755
- S.2.9. Legendre Functions and Legendre Polynomials p. 756
- S.2.10. Parabolic Cylinder Functions p. 758
- S.2.11. Orthogonal Polynomials p. 759
- S.2.12. The Weierstrass Function p. 762
- S.3. Tables of Indefinite Integrals p. 763
- S.3.1. Integrals Containing Rational Functions p. 763
- S.3.2. Integrals Containing Irrational Functions p. 767
- S.3.3. Integrals Containing Exponential Functions p. 769
- S.3.4. Integrals Containing Hyperbolic Functions p. 769
- S.3.5. Integrals Containing Logarithmic Functions p. 772
- S.3.6. Integrals Containing Trigonometric Functions p. 773
- S.3.7. Integrals Containing Inverse Trigonometric Functions p. 777
- References p. 779
- Index p. 783