

- Foreword p. xiii
- Part I Mechanical behaviour of composite materials
- 1 Constitutive relations for anisotropic materials in linear elasticity p. 3
 - 1.1 Introduction p. 3
 - 1.2 Four indices tensor notation p. 3
 - 1.2.1 Constitutive relations p. 3
 - 1.2.2 Properties of C_{ijkl} and S_{ijkl} p. 4
- 1.3 Conventional two indices Voigt notation p. 5
- 1.4 Anisotropic material p. 6
 - 1.4.1 Monoclinic material p. 6
 - 1.4.2 Orthotropic material p. 9
 - 1.4.3 Transversely isotropic material p. 14
 - 1.4.4 Isotropic material p. 17
 - 1.4.5 Influence of temperature and humidity on the constitutive relation of an orthotropic material p. 19
- 1.5 Matrix relations for a change of axes p. 20
 - 1.5.1 Change of axes for stress and strain matrices p. 20
 - 1.5.2 Change of axes for stiffness and compliance matrices p. 24
- 2 Orthotropic layer behaviour p. 25
 - 2.1 Introduction p. 25
 - 2.2 Stiffness and compliance matrices in orthotropic co-ordinates p. 25
 - 2.3 Conventional matrices for changing axes p. 27
 - 2.4 Stress and strain matrices p. 29
 - 2.5 Stiffness matrix in directions away from the orthotropic axes p. 30
 - 2.6 Compliance matrix in directions away from the orthotropic axes p. 32
 - 2.7 Orthotropic layer loaded in tension and in shear p. 35
 - 2.7.1 Simple tension p. 35
 - 2.7.2 Simple shear p. 37
 - 2.8 Reduced stiffness matrix for the orthotropic layer p. 38
 - 2.8.1 Reduced stiffness matrix Q in orthotropic co-ordinates p. 38
 - 2.8.2 Reduced stiffness matrix Q in the co-ordinates away from the orthotropic axes p. 40
- 2.9 Reduced compliance matrices of an orthotropic layer p. 43
 - 2.9.1 Reduced compliance matrix in orthotropic co-ordinates p. 43
 - 2.9.2 Reduced compliance matrix P in the direction away from the orthotropic axes p. 43
- 3 Elastic constants of a unidirectional composite p. 45
 - 3.1 Introduction p. 45
 - 3.2 Density ρ p. 45
 - 3.3 Longitudinal Young's modulus E_1 p. 46
 - 3.4 Poisson's coefficient ν_{12} p. 47
 - 3.5 Transverse Young's modulus E_2 p. 48
 - 3.6 Shear modulus G_{12} p. 49
 - 3.7 Longitudinal thermal expansion coefficient α_1 p. 51
 - 3.8 Transverse expansion coefficient α_2 p. 52
- 4 Failure criteria p. 55

- 4.1 Introduction p. 55
- 4.2 Maximum stress theory p. 55
- 4.3 Maximum strain theory p. 56
- 4.4 Polynomial failure criteria p. 57
 - 4.4.1 Tsai-Hill criterion p. 57
 - 4.4.2 Tsai-Wu criterion p. 60
 - 4.4.3 Hoffman criterion p. 62
- 4.5 Tensile and shear strength of a unidirectional layer p. 64
 - 4.5.1 Tensile strength p. 64
 - 4.5.2 Shear strength p. 67
- 4.6 Determination of failure stresses from three tension tests p. 70
- Part II Multi-layer plates
- 5 Multi-layer Kirchhoff-Love thin plates p. 73
 - 5.1 Introduction p. 73
 - 5.2 Kirchhoff-Love hypotheses for thin plates p. 73
 - 5.3 Strain-displacement relationships p. 74
 - 5.4 Global plate equations p. 75
 - 5.5 Calculation of $I_{[0]}$ p. 76
 - 5.6 Stress field p. 77
 - 5.7 Global cohesive forces p. 77
 - 5.8 Composite global stiffness matrix p. 78
 - 5.9 Decoupling p. 79
 - 5.9.1 Membrane-flexion decoupling p. 79
 - 5.9.2 Tension-shear decoupling p. 80
 - 5.9.3 Membrane-flexion and tension-shear decoupling p. 82
 - 5.10 Global stiffnesses of a symmetrical composite p. 82
 - 5.10.1 Symmetrical laminate $([\alpha],[\beta])_{[NS]}$ p. 82
 - 5.10.2 Symmetrical cross-ply laminate $(0,[\pi]/2)_{[NS]}$ p. 85
 - 5.10.3 Symmetrical balanced laminate $([\alpha],[-\alpha])_{[NS]}$ p. 86
 - 5.11 Global stiffnesses for an asymmetrical laminate p. 87
 - 5.11.1 Asymmetrical laminate $([\alpha],[\beta])_{[N]}$ p. 87
 - 5.11.2 Asymmetrical cross-ply laminate $(0,[\pi]/2)_{[N]}$ p. 90
 - 5.11.3 Asymmetrical balanced laminate $([\alpha],-[\alpha])_{[N]}$ p. 91
 - 5.12 Examples of global stiffness matrices p. 91
 - 5.12.1 Two layer plate p. 91
 - 5.12.2 Three layer plate p. 92
 - 5.12.3 Four layer plate p. 93
 - 5.12.4 Examples of decoupling p. 93
 - 5.13 Boundary conditions p. 95
 - 5.13.1 Definition of boundary conditions p. 95
 - 5.13.2 Effective global transverse shear load p. 96
 - 5.14 Determination of transverse shear stresses p. 100
 - 5.15 Strain energy p. 103
 - 6 Symmetrical orthotropic Kirchhoff-Love plates p. 105
 - 6.1 Introduction p. 105
 - 6.2 Global plate equations p. 106

- 6.3 Plate loaded in the mean plane p. 106
- 6.4 Plate loaded transversely p. 107
- 6.5 Flexure of a rectangular plate simply supported around its edge p. 109
- 6.6 Free vibrations of a rectangular plate freely supported at its edge p. 118
- 6.7 Buckling of a rectangular plate simply supported at its edge p. 121
 - 6.7.1 General case p. 121
 - 6.7.2 Case of $k = 0$ p. 123
 - 6.7.3 Case of $k = 1$ p. 127
 - 6.7.4 Case of $k = -1/2$ p. 128
- 7 Thermo-elastic behaviour of composites p. 131
 - 7.1 Introduction p. 131
 - 7.2 Constitutive relation for an orthotropic material p. 131
 - 7.2.1 Constitutive relation in orthotropic axes p. 131
 - 7.2.2 Constitutive relation in orthotropic off-axes p. 133
 - 7.3 Constitutive relation when the normal transverse stress is zero p. 135
 - 7.3.1 Constitutive relation in orthotropic axes p. 135
 - 7.3.2 Constitutive relation in orthotropic off-axes p. 135
 - 7.4 Global cohesion forces p. 136
 - 7.5 Global composite constitutive relation p. 137
 - 7.6 Decoupling p. 138
 - 7.6.1 Composite with mirror symmetry p. 138
 - 7.6.2 Balanced composite p. 138
 - 7.6.3 Balanced symmetrical composite p. 139
 - 7.7 Balanced symmetrical composite loaded in the mean plane p. 139
- 8 Symmetrical orthotropic Reissner-Mindlin plates p. 141
 - 8.1 Introduction p. 141
 - 8.2 Moderately thick plate, Reissner-Mindlin assumptions p. 141
 - 8.3 Displacements, strains and stresses p. 141
 - 8.4 Global plate equations p. 142
 - 8.5 Calculation of $I_{[1]}$ and $I_{[2]}$ p. 142
 - 8.6 Global cohesive forces p. 143
 - 8.7 Global stiffness matrix of the composite p. 144
 - 8.8 Transverse shear correction coefficient p. 145
 - 8.8.1 Uflyand coefficient p. 145
 - 8.8.2 Reissner coefficient p. 147
 - 8.9 Boundary conditions p. 149
 - 8.10 Symmetrical orthotropic plate p. 149
 - 8.11 Flexure of a rectangular orthotropic symmetrical plate simply supported around its edge p. 150
 - 8.12 Transverse vibration of a rectangular orthotropic symmetrical plate simply supported around its edge p. 153
 - 8.13 Buckling of a rectangular orthotropic symmetrical plate simply supported around its edge p. 155
- 9 Asymmetrical multi-layer Kirchhoff-Love plates p. 159
 - 9.1 Introduction p. 159
 - 9.2 Flexure of a cross-ply asymmetrical plate p. 159

- 9.3 Vibration of an cross-ply asymmetrical plate p. 165
- 9.4 Buckling of a cross-ply asymmetrical plate p. 170
- 9.5 Flexure of a balanced asymmetrical plate p. 175
- 9.6 Vibration of a balanced asymmetrical plate p. 182
- 9.7 Buckling of a balanced asymmetrical plate p. 189
- 10 Cylindrical flexure of multi-layer Kirchhoff-Love plates p. 195
- 10.1 Introduction p. 195
- 10.2 Strain-displacement relationship p. 195
- 10.3 Global constitutive relation p. 196
- 10.4 Global plate equations p. 197
- 10.5 Flexure p. 198
- 10.5.1 Elimination of u^0_1 and u^0_2 p. 199
- 10.5.2 Simply supported plate subjected to a sinusoidal load p. 200
- 10.6 Vibrations p. 204
- 10.6.1 General case p. 204
- 10.6.2 Asymmetrical cross-ply composite $(0,[\pi]/2)[N]$ p. 206
- 10.6.3 Balanced asymmetrical composite $([\alpha],-[\alpha])[N]$ p. 208
- 10.6.4 Symmetrical composite p. 209
- 10.7 Buckling p. 211
- 10.7.1 General case p. 211
- 10.7.2 Cross-ply asymmetrical composite $(0,[\pi]/2)[N]$ p. 214
- 10.7.3 Balanced asymmetrical composite $([\alpha],-[\alpha])[N]$ p. 214
- 10.7.4 Symmetrical composite p. 214
- 11 Cylindrical flexure of multi-layer Reissner-Mindlin plates p. 215
- 11.1 Introduction p. 215
- 11.2 Strain-displacement relationship p. 215
- 11.3 Global constitutive relation p. 215
- 11.4 Global plate equations p. 216
- 11.5 Flexure p. 218
- 11.6 Vibrations p. 222
- 11.7 Buckling p. 225
- Part III Multi-layer beams
- 12 Symmetrical multi-layer beams in tension-compression p. 231
- 12.1 Introduction p. 231
- 12.2 Strains, stresses, global equation of tension-compression p. 231
- 12.3 Single layer orthotropic beam p. 233
- 12.4 General equations for beams in tension-compression p. 234
- 12.5 Built-in beam under its own weight and subjected to a force p. 235
- 12.6 Vibration of a built-in beam p. 237
- 13 Symmetrical multi-layer beams in flexure without transverse shear strain p. 239
- 13.1 Introduction p. 239
- 13.2 Strains, stresses, equations of motion p. 239
- 13.3 Monolayer orthotropic beam p. 243
- 13.4 General beam equations p. 244
- 13.4.1 Flexure in the plane $(O[x_1,x_3])$ p. 244
- 13.4.2 Flexure in the plane $(O[x_1,x_2])$ p. 247

- 13.5 Simply supported beam subjected to sinusoidal loads p. 249
- 13.6 Vibrations of a simply supported beam p. 250
- 13.7 Buckling of a simply supported beam p. 251
- 14 Symmetrical multi-layer beams in flexure with transverse shear strain p. 253
- 14.1 Introduction p. 253
- 14.2 Strains, stresses, global equations p. 253
- 14.3 Monolayer orthotropic beam p. 256
- 14.4 General beam equations p. 258
- 14.4.1 Flexure in the plane ($O[x_1, x_3]$) p. 258
- 14.4.2 Flexure in the plane ($O[x_1, x_2]$) p. 260
- 14.5 Simply supported beam subjected to a sinusoidal load p. 261
- 14.6 Vibration of a simply supported beam p. 263
- 14.7 Buckling of a simply supported beam p. 266
- Appendices Global plate equations
- 15 Global plate equations neglecting large transverse displacements p. 271
- 15.1 Introduction p. 271
- 15.2 Hypothesis relating to plates p. 271
- 15.3 Reissner-Mindlin and Kirchhoff-Love plates theories p. 272
- 15.4 Global plate equations p. 278
- 15.4.1 Force equations p. 278
- 15.4.2 Moment equations p. 280
- 15.5 Plate equations in Reissner-Mindlin analysis p. 281
- 15.5.1 Calculation of second members p. 281
- 15.5.2 Global plate equations p. 282
- 15.5.3 Boundary conditions p. 283
- 15.6 Plate equations in Kirchhoff-Love analysis p. 283
- 15.6.1 Calculation of second members p. 283
- 15.6.2 Global plate equations p. 284
- 15.6.3 Boundary edge conditions p. 285
- 16 Global plate equations for large transverse displacements p. 287
- 16.1 Introduction p. 287
- 16.2 Local plate equations p. 287
- 16.3 Global plate equations p. 288
- 16.3.1 Global plate summation equations p. 288
- 16.3.2 Global plate moment equations p. 291
- 16.4 Global plate equations for static, vibration and buckling cases p. 293
- 16.4.1 Global plate equations p. 293
- 16.4.2 Global plate equilibrium equations p. 293
- 16.4.3 Global plate vibration equations p. 294
- 16.4.4 Global plate buckling equations p. 295
- 16.5 Reissner-Mindlin global plate equations p. 295
- 16.6 Kirchhoff-Love global plate equations p. 296
- 17 Global plate equations: Kirchhoff-Love theory variational formulation p. 297
- 17.1 Introduction p. 297
- 17.2 Von Karman strains p. 297
- 17.3 Variational formulation p. 301

- 17.3.1 Virtual work of internal forces p. 301
- 17.3.2 Virtual work of transverse surface forces p. 304
- 17.3.3 Virtual work of external lateral surface forces p. 304
- 17.3.4 Virtual work of body forces p. 305
- 17.3.5 Virtual work done by inertial forces p. 306
- 17.3.6 Variational formulation p. 307
- 17.4 Global plate equations, boundary edge conditions p. 308
- 17.4.1 Global plate equations p. 308
- 17.4.2 Boundary edge conditions p. 309
- 17.5 Global plate equations in static, vibration and buckling cases p. 313
- 18 Global plate equations: Reissner-Mindlin theory variational formulation p. 315
- 18.1 Introduction p. 315
- 18.2 Von Karman strains p. 315
- 18.3 Variational formulation p. 316
- 18.3.1 Virtual work of internal forces p. 317
- 18.3.2 Virtual work of transverse surface forces p. 320
- 18.3.3 Virtual work of external lateral surface forces p. 320
- 18.3.4 Virtual work of body forces p. 320
- 18.3.5 Virtual work done by inertial forces p. 321
- 18.3.6 Variational formulation p. 322
- 18.4 Global equations, boundary edge conditions p. 323
- 18.4.1 Global plate equations p. 323
- 18.4.2 Boundary edge conditions p. 325
- 18.5 Global static, vibration and buckling equations p. 328
- References p. 331
- Index p. 333