

Contents

Preface	xvii	
About the Author	xix	
Trademarks	xx	
Chapter 1	What is Operations Research?	1
1.1	Operations Research Models	1
1.2	Solving the OR Model	4
1.3	Queuing and Simulation Models	5
1.4	Art of Modeling	5
1.5	More Than Just Mathematics	6
1.6	Phases of an OR Study	8
1.7	About This Book	9
References	9	
Chapter 2	Modeling with Linear Programming	10
2.1	Two-Variable LP Model	11
2.2	Graphical LP Solution	16
2.2.1	Solution of a Maximization Model	16
2.2.2	Solution of a Minimization Model	22
2.3	Selected LP Applications	27
2.3.1	Urban Planning	28
2.3.2	Currency Arbitrage	31
2.3.3	Investment	36
2.3.4	Production Planning and Inventory Control	40
2.3.5	Blending and Refining	48
2.3.6	Manpower Planning	52
2.3.7	Additional Applications	55
2.4	Computer Solution with Solver and AMPL	63
2.4.1	LP Solution with Excel Solver	63
2.4.2	LP Solution with AMPL	67
References	74	
Chapter 3	The Simplex Method and Sensitivity Analysis	75
3.1	LP Model in Equation Form	76
3.1.1	Converting Inequalities into Equations with Nonnegative Right-Hand Side	76
3.1.2	Dealing with Unrestricted Variables	78
3.2	Transition from Graphical to Algebraic Solution	79

3.3 The Simplex Method	85
3.3.1 Iterative Nature of the Simplex Method	85
3.3.2 Computational Details of the Simplex Algorithm	87
3.3.3 Summary of the Simplex Method	95
3.4 Artificial Starting Solution	99
3.4.1 M-Method	99
3.4.2 Two-Phase Method	104
3.5 Special Cases in the Simplex Method	109
3.5.1 Degeneracy	109
3.5.2 Alternative Optima	112
3.5.3 Unbounded Solution	115
3.5.4 Infeasible Solution	117
3.6 Sensitivity Analysis	119
3.6.1 Graphical Sensitivity Analysis	119
3.6.2 Algebraic Sensitivity Analysis—Changes in the Right-Hand Side	125
3.6.3 Algebraic Sensitivity Analysis—Objective Function	133
3.6.4 Sensitivity Analysis with TORA, Solver, and AMPL	139
References	142

Chapter 4 Duality and Post-Optimal Analysis **143**

4.1 Definition of the Dual Problem	143
4.2 Primal-Dual Relationships	148
4.2.1 Review of Simple Matrix Operations	148
4.2.2 Simplex Tableau Layout	150
4.2.3 Optimal Dual Solution	151
4.2.4 Simplex Tableau Computations	157
4.3 Economic Interpretation of Duality	160
4.3.1 Economic Interpretation of Dual Variables	160
4.3.2 Economic Interpretation of Dual Constraints	162
4.4 Additional Simplex Algorithms	164
4.4.1 Dual Simplex Algorithm	164
4.4.2 Generalized Simplex Algorithm	171
4.5 Post-Optimal Analysis	172
4.5.1 Changes Affecting Feasibility	173
4.5.2 Changes Affecting Optimality	178
References	182

Chapter 5 Transportation Model and Its Variants **183**

5.1 Definition of the Transportation Model	184
5.2 Nontraditional Transportation Models	188
5.3 The Transportation Algorithm	193
5.3.1 Determination of the Starting Solution	194
5.3.2 Iterative Computations of the Transportation Algorithm	198

Chapter 5	5.3.3 Simplex Method Explanation of the Method of Multipliers 210
5.4	The Assignment Model 211
	5.4.1 The Hungarian Method 212
	5.4.2 Simplex Explanation of the Hungarian Method 219
5.5	The Transshipment Model 220
	References 224

Chapter 6 Network Models 225

6.1	Scope and Definition of Network Models 226
6.2	Minimal Spanning Tree Algorithm 229
6.3	Shortest-Route Problem 233
	6.3.1 Examples of the Shortest-Route Applications 233
	6.3.2 Shortest-Route Algorithms 236
	6.3.3 Linear Programming Formulation of the Shortest-Route Problem 246
6.4	Maximal flow model 251
	6.4.1 Enumeration of Cuts 252
	6.4.2 Maximal-Flow Algorithm 253
	6.4.3 Linear Programming Formulation of Maximal Flow Mode 260
6.5	CPM and PERT 262
	6.5.1 Network Representation 264
	6.5.2 Critical Path (CPM) Computations 269
	6.5.3 Construction of the Time Schedule 272
	6.5.4 Linear Programming Formulation of CPM 278
	6.5.5 PERT Calculations 279
	References 285

Chapter 7 Advanced Linear Programming 287

7.1	Simplex Method Fundamentals 288
	7.1.1 From Extreme Points to Basic Solutions 289
	7.1.2 Generalized Simplex Tableau in Matrix Form 293
7.2	Revised Simplex Method 296
	7.2.1 Development of the Optimality and Feasibility Conditions 296
	7.2.2 Revised Simplex Algorithm 299
7.3	Bounded-Variables Algorithm 306
7.4	Duality 311
	7.4.1 Matrix Definition of the Dual Problem 311
	7.4.2 Optimal Dual Solution 312
7.5	Parametric Linear Programming 316
	7.5.1 Parametric Changes in C 316
	7.5.2 Parametric Changes in b 319
	References 322

Chapter 8	Goal Programming	323
8.1	A Goal Programming Formulation	324
8.2	Goal Programming Algorithms	327
8.2.1	The Weights Method	328
8.2.2	The Preemptive Method	330
	References	337
Chapter 9	Integer Linear Programming	338
9.1	Illustrative Applications	339
9.1.1	Capital Budgeting	339
9.1.2	Set-Covering Problem	343
9.1.3	Fixed-Charge Problem	347
9.1.4	Either-Or and If-Then Constraints	351
9.2	Integer Programming Algorithms	354
9.2.1	Branch-and-Bound (B&B) Algorithm	355
9.2.2	Cutting-Plane Algorithm	362
9.2.3	Computational Considerations in ILP	367
9.3	Traveling Salesperson Problem (TSP)	368
9.3.1	Heuristic Algorithms	371
9.3.2	B&B Solution Algorithm	374
9.3.3	Cutting-Plane Algorithm	377
	References	379
Chapter 10	Deterministic Dynamic Programming	380
10.1	Recursive Nature of Computations in DP	381
10.2	Forward and Backward Recursion	384
10.3	Selected DP Applications	386
10.3.1	Knapsack/Fly-Away/Cargo-Loading Model	386
10.3.2	Work-Force Size Model	399
10.3.3	Equipment Replacement Model	401
10.3.4	Investment Model	405
10.3.5	Inventory Models	408
10.4	Problem of Dimensionality	408
	References	411
Chapter 11	Deterministic Inventory Models	412
11.1	General Inventory Model	412
11.2	Role of Demand in the Development of Inventory Models	413
11.3	Static Economic-Order-Quantity (EOQ) Models	415
11.3.1	Classic EOQ model	415
11.3.2	EOQ Problems with Price Breaks	421
11.3.3	Multi-Item EOQ with Storage Limitation	428
11.4	Dynamic EOQ Models	432
11.4.1	No-Setup Model	433
11.4.2	Setup Model	437
	References	448

Chapter 12 Review of Basic Probability 449

12.1	Laws of Probability	449
12.1.1	Addition Law of Probability	450
12.1.2	Conditional Law of Probability	452
12.2	Random Variables and Probability Distributions	453
12.3	Expectation of a Random Variable	455
12.3.1	Mean and Variance (Standard Deviation) of a Random Variable	456
12.3.2	Mean and Variance of Joint Random Variable	458
12.4	Four Common Probability Distributions	461
12.4.1	Binomial Distribution	461
12.4.2	Poisson Distribution	462
12.4.3	Negative Exponential Distribution	463
12.4.4	Normal Distribution	464
12.5	Empirical Distributions	467
	References	475

Chapter 13 Decision Analysis and Games 476

13.1	Decision Making under Certainty—Analytic Hierarchy Process (AHP)	477
13.2	Decision Making under Risk	486
13.2.1	Decision Tree-Based Expected Value Criterion	486
13.2.2	Variations of the Expected Value Criterion	492
13.3	Decision under Uncertainty	500
13.4	Game Theory	505
13.4.1	Some Basic Terminologies	506
13.4.2	Optimal Solution of Two-Person Zero-Sum Games	507
13.4.3	Solution of Mixed Strategy Games	510
	References	521

Chapter 14 Probabilistic Inventory Models 522

14.1	Continuous Review Models	523
14.1.1	"Probabilitized" EOQ Model	523
14.1.2	Probabilistic EOQ Model	525
14.2	Single-Period Models	529
14.2.1	No-Setup Model (Newsvendor Model)	530
14.2.2	Setup Model ($s-S$ Policy)	533
14.3	Multiperiod Model	536
	References	538

Chapter 15 Queuing Systems 539

15.1	Why Study Queues?	540
15.2	Elements of a Queuing Model	541
15.3	Role of Exponential Distribution	543
15.4	Pure Birth and Death Models (Relationship Between the Exponential and Poisson Distributions)	546
15.4.1	Pure Birth Model	546
15.4.2	Pure Death Model	550

Chapter 15	Queuing Models	551
15.5	Generalized Poisson Queuing Model	552
15.6	Specialized Poisson Queues	557
15.6.1	Steady-State Measures of Performance	558
15.6.2	Single-Server Models	561
15.6.3	Multiple-Server Models	572
15.6.4	Machine Servicing Model—(M/M/R): (GD/K/K), $R < K$	581
15.7	(M/G/1):(GD/ ∞/∞)—Pollaczek-Khintchine (P-K) Formula	584
15.8	Other Queuing Models	587
15.9	Queuing Decision Models	587
15.9.1	Cost Models	587
15.9.2	Aspiration Level Model	592
	References	593
Chapter 16	Simulation Modeling	594
16.1	Monte Carlo Simulation	594
16.2	Types of Simulation	599
16.3	Elements of Discrete-Event Simulation	599
16.3.1	Generic Definition of Events	599
16.3.2	Sampling from Probability Distributions	601
16.4	Generation of Random Numbers	609
16.5	Mechanics of Discrete Simulation	611
16.5.1	Manual Simulation of a Single-Server Model	611
16.5.2	Spreadsheet-Based Simulation of the Single-Server Model	616
16.6	Methods for Gathering Statistical Observations	619
16.6.1	Subinterval Method	620
16.6.2	Replication Method	621
16.6.3	Regenerative (Cycle) Method	622
16.7	Simulation Languages	624
	References	626
Chapter 17	Markov Chains	627
17.1	Definition of a Markov Chain	627
17.2	Absolute and n -Step Transition Probabilities	629
17.3	Classification of the States in a Markov Chain	631
17.4	Steady-State Probabilities and Mean Return Times of Ergodic Chains	633
17.5	First Passage Time	639
17.6	Analysis of Absorbing States	642
	References	647
Chapter 18	Classical Optimization Theory	648
18.1	Unconstrained Problems	648
18.1.1	Necessary and Sufficient Conditions	649
18.1.2	The Newton-Raphson Method	653

18.2 Constrained Problems	655
18.2.1 Equality Constraints	656
18.2.2 Inequality Constraints—Karush-Kuhn-Tucker (KKT) Conditions	668
References	672

Chapter 19 Nonlinear Programming Algorithms **673**

19.1 Unconstrained Algorithms	673
19.1.1 Direct Search Method	673
19.1.2 Gradient Method	677
19.2 Constrained Algorithms	680
19.2.1 Separable Programming	681
19.2.2 Quadratic Programming	690
19.2.3 Chance-Constrained Programming	694
19.2.4 Linear Combinations Method	699
19.2.5 SUMT Algorithm	701
References	702

Appendix A AMPL Modeling Language **703**

A.1 Rudimentary AMPL Model	703
A.2 Components of AMPL Model	704
A.3 Mathematical Expressions and Computed Parameters	712
A.4 Subsets and Indexed Sets	715
A.5 Accessing External Files	717
A.5.1 Simple Read Files	717
A.5.2 Using Print or Printf to Retrieve Output	718
A.5.3 Input Table Files	719
A.5.4 Output Table Files	721
A.5.5 Spreadsheet Input/Output Tables	723
A.6 Interactive Commands	724
A.7 Iterative and Conditional Execution of AMPL Commands	725
A.8 Sensitivity Analysis Using AMPL	727
Reference	727

Appendix B Statistical Tables **728**

Appendix C Partial Answers to Selected Problems **733**

Index **765**

Chapter 26 Case Analyses **CD-79**

- Case 1: Airline Fuel Allocation Using Optimum Scheduling — CD-79
- Case 2: Optimization of Heavy-Vehicle Production — CD-86
- Case 3: Scheduling Arrangements of Australian Tourist Commission Trade Events — CD-93